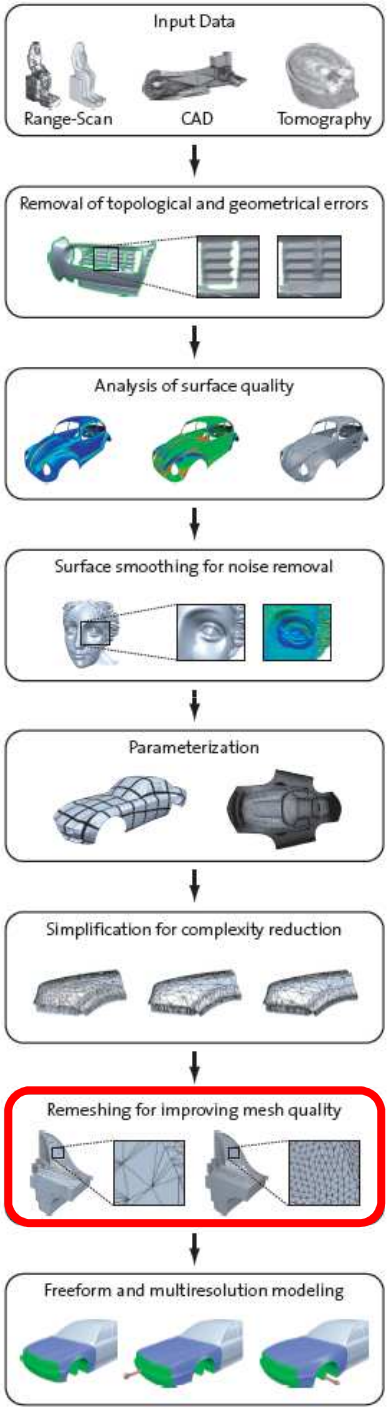

Remeshing

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DE RECHERCHE
EN INFORMATIQUE
ET EN AUTOMATIQUE

INRIA

centre de recherche SOPHIA ANTIPOLIS - MÉDITERRANÉE

SIGGRAPH2007

Definition

Remeshing:

Definition

Remeshing:

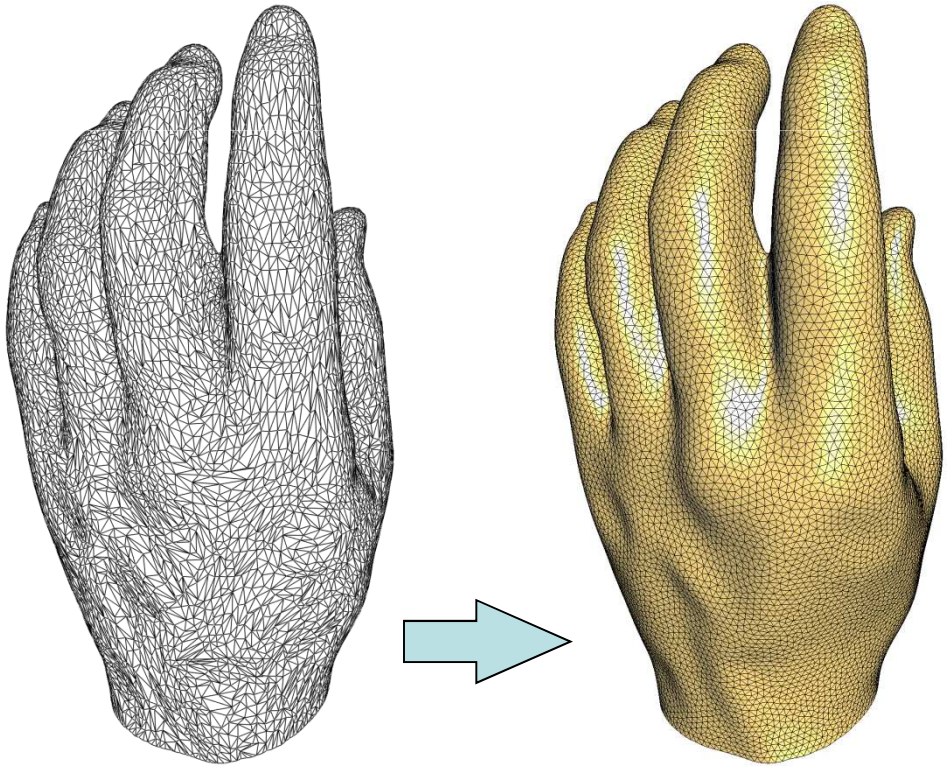
- partitioning a domain with elements that meet at edges.
- control/optimization over:
  - #elements
  - shape
  - orientation
  - size
  - grading

= discretization

Definition

Remeshing:

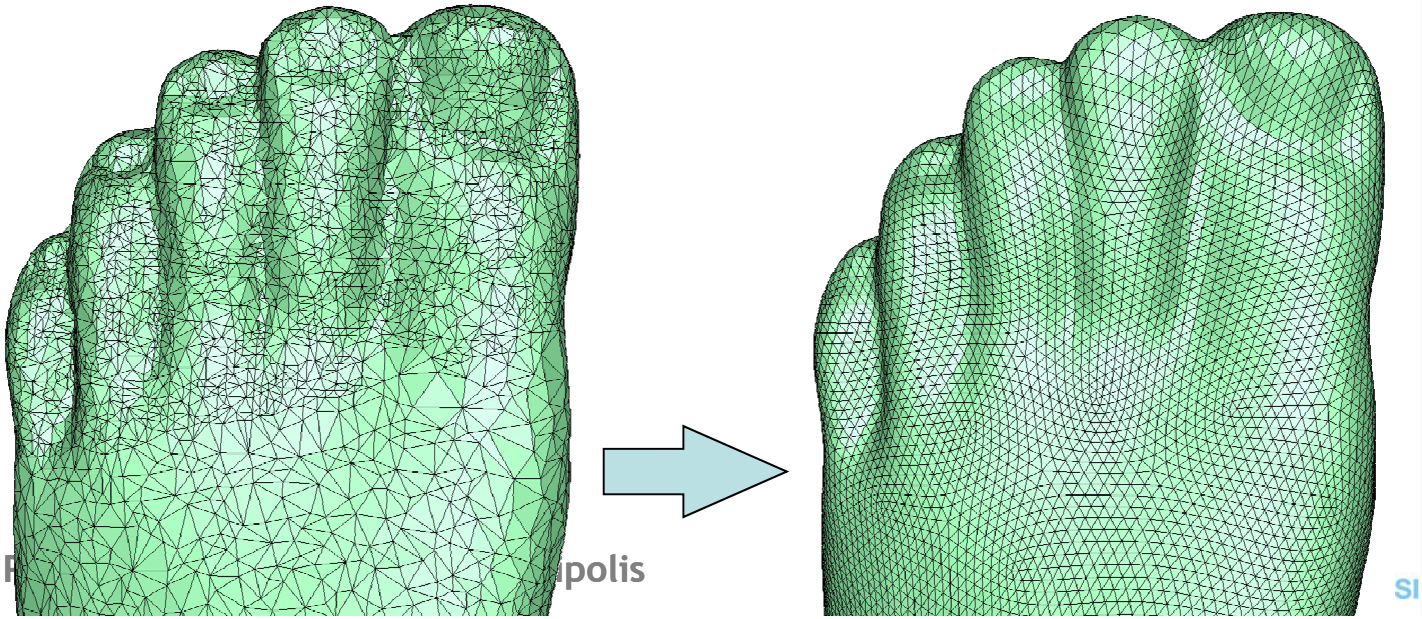
- Altering the mesh geometry and connectivity to improve quality.



Definition

Remeshing:

- Altering the mesh geometry and connectivity to improve quality.
- Replacing an arbitrarily structured mesh by a structured one.



Definition

Remeshing:

- Altering the mesh geometry and connectivity to improve quality.
- Replacing an arbitrarily structured mesh by a structured one.

...while approximating well the input

Applications

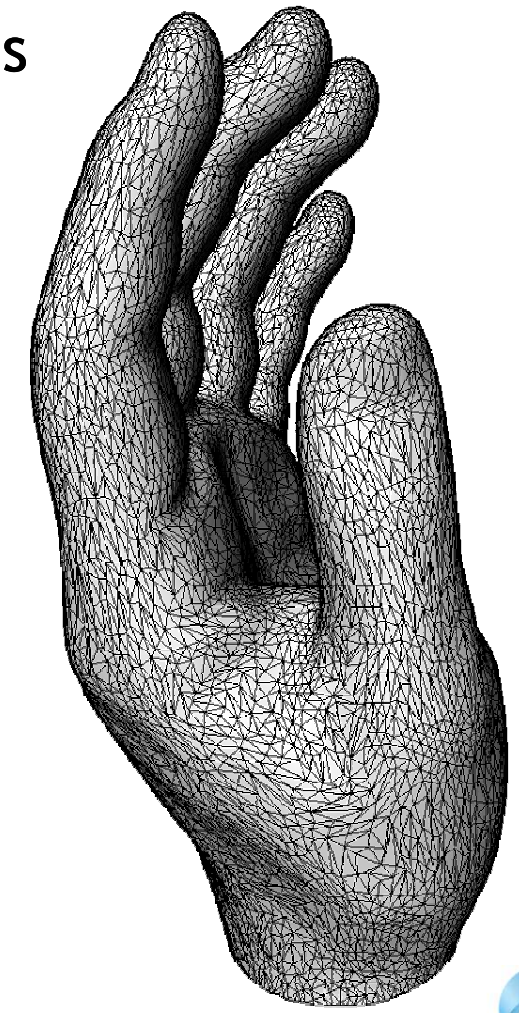
Used in various fields:

- Design
- Reverse engineering
- Simulation
- Visualization

Remeshing

for scanned geometry:

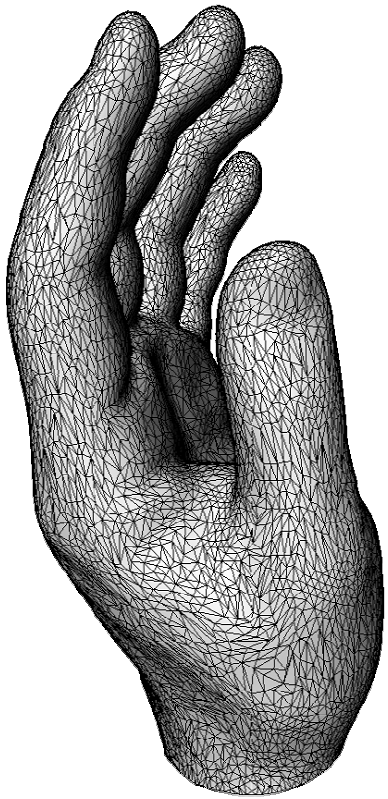
- The (approximation of a) surface is given by a triangle mesh:
  - non uniform
  - irregular
  - arbitrary genus
- goal: **improve mesh quality**



Remeshing

for simulation and geometry processing

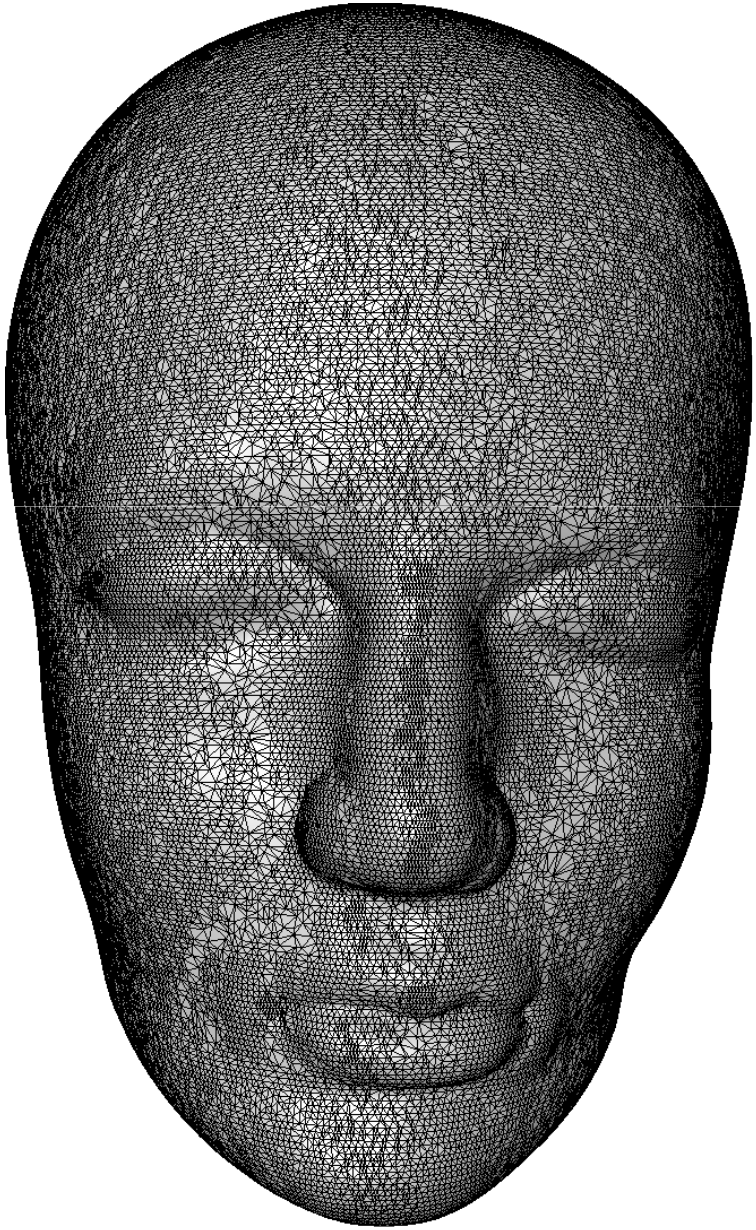
- most scanned surfaces need remeshing to improve:
  - size
  - sampling
  - grading
  - regularity
  - shape of elements



Remeshing

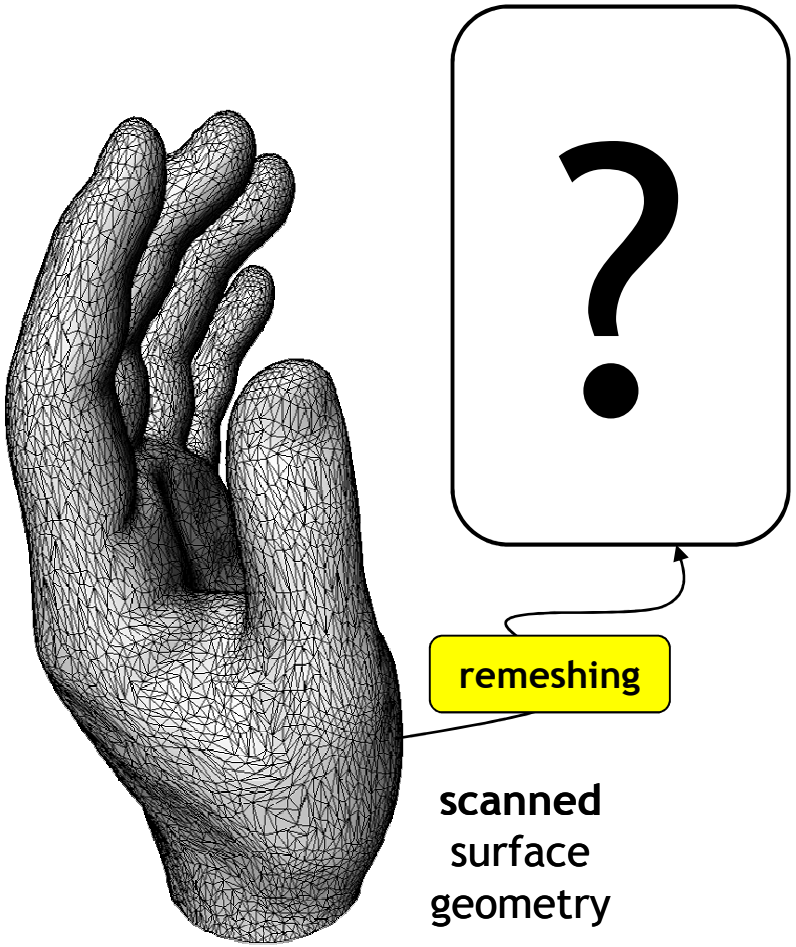
For **efficient** shape modeling

3D datasets are often overly verbose. Visualization and processing requires concise, yet faithful representations.

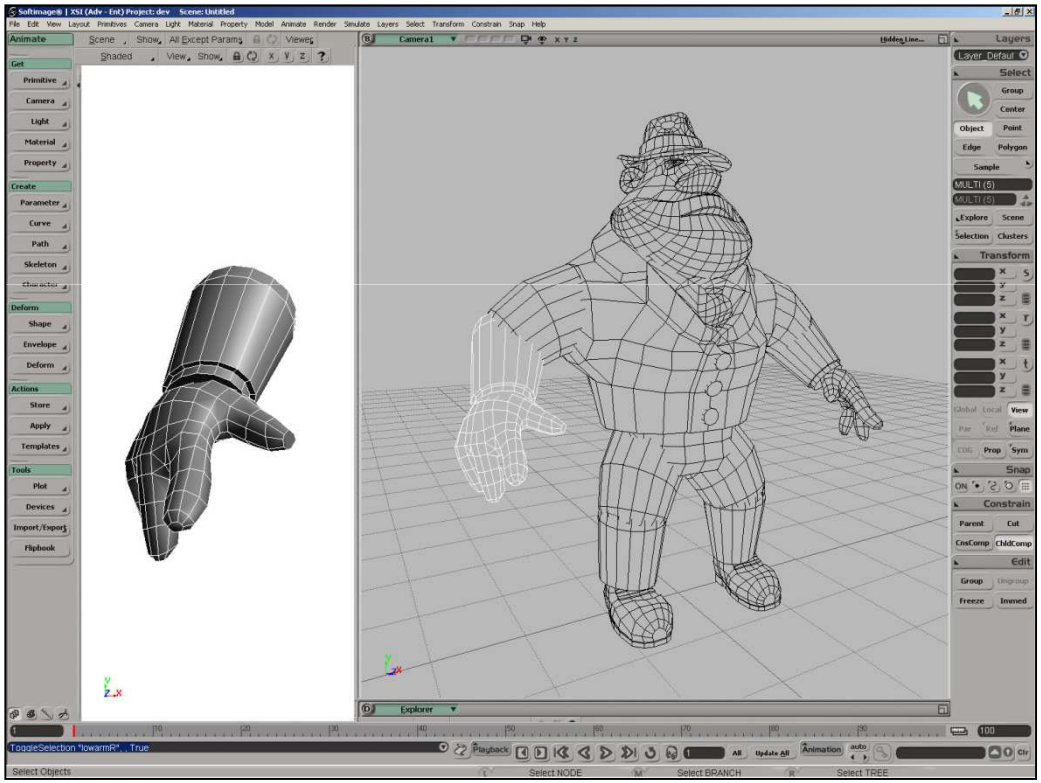


Remeshing

for **efficient** shape modeling



scanned
surface
geometry

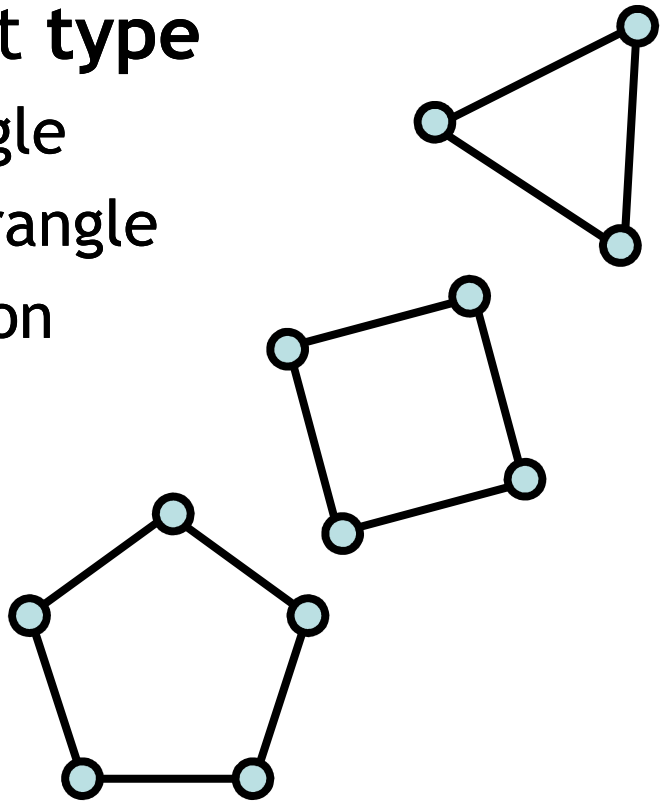


carefully designed model

www.viewpoint.com

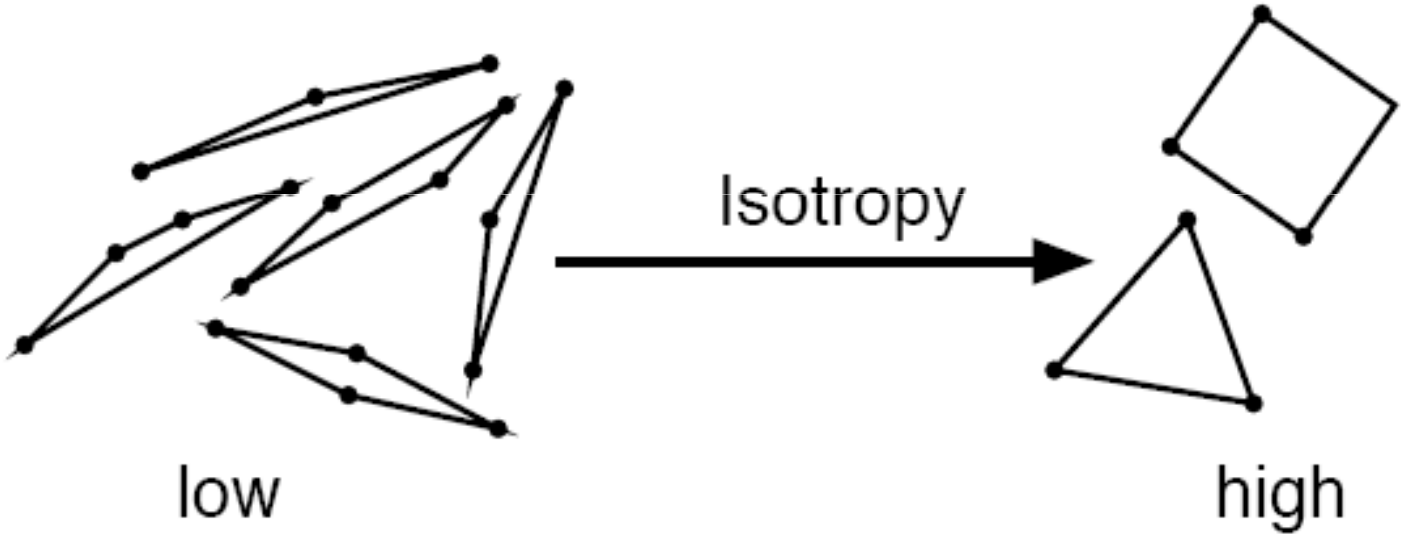
Local Structure

- Element **type**
  - Triangle
  - Quadrangle
  - Polygon



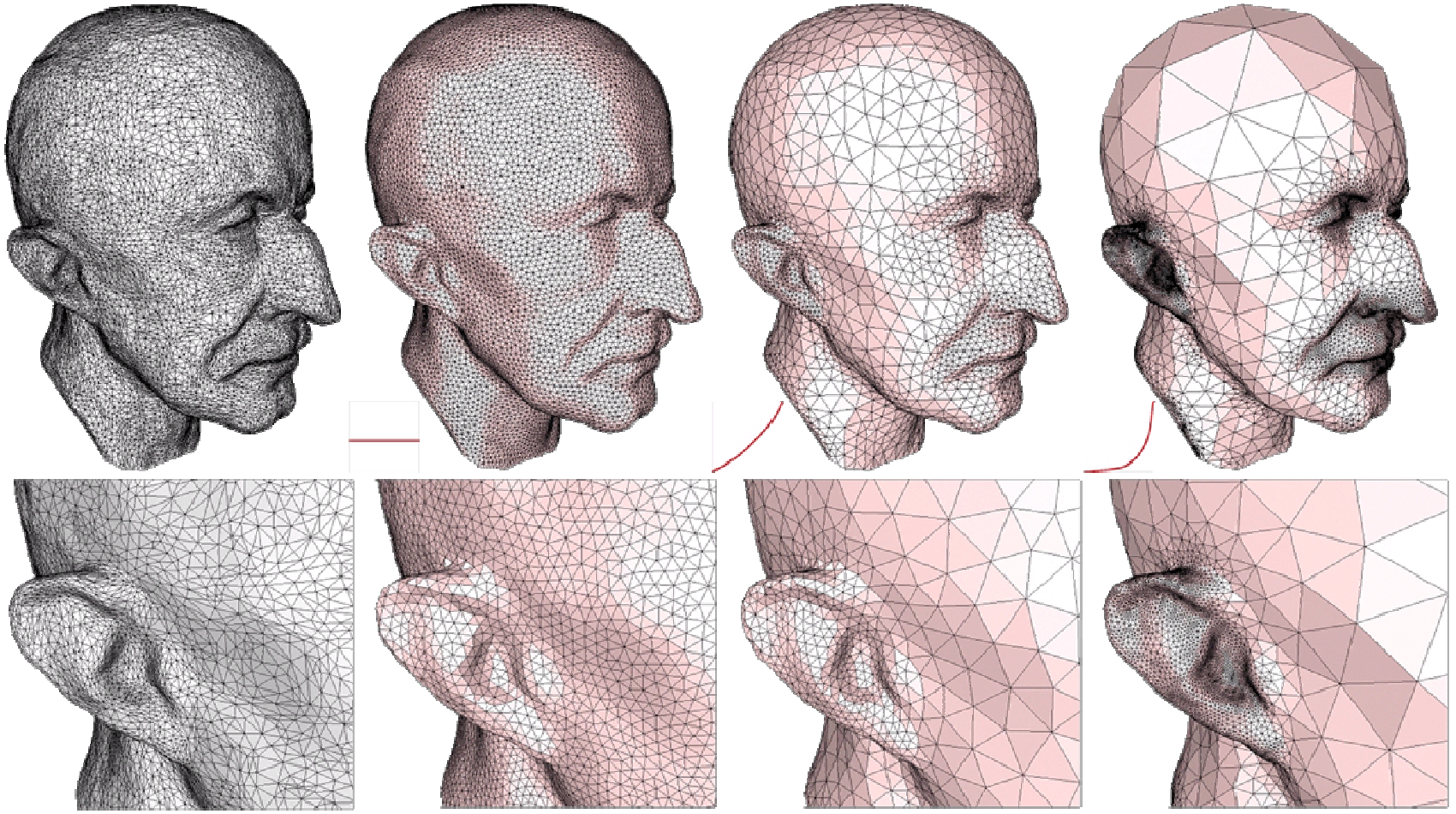
Local Structure

- Element **shape** (isotropy vs anisotropy)



Local Structure

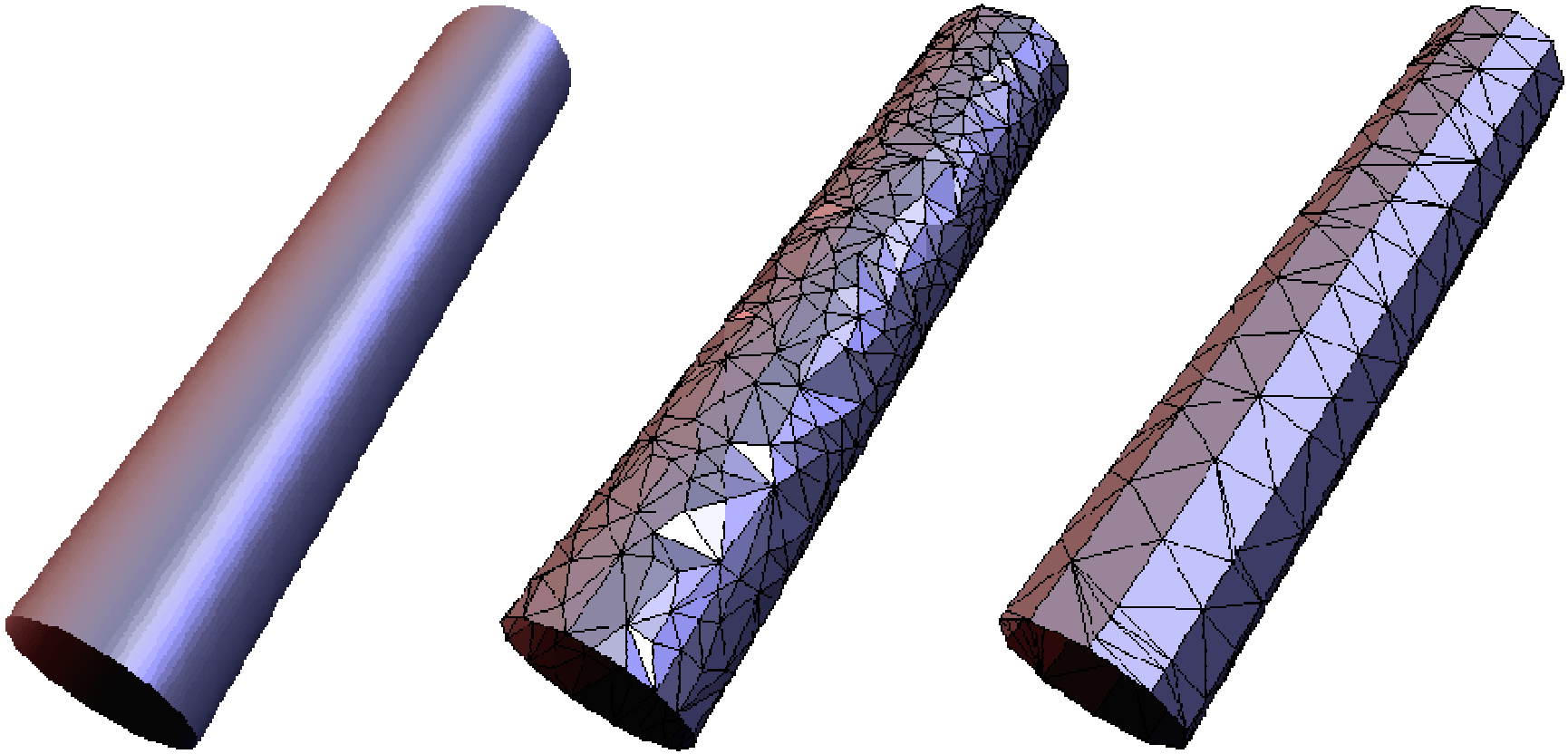
Element distribution (**sizing, grading**)



Input **Uniform** **Adapted**

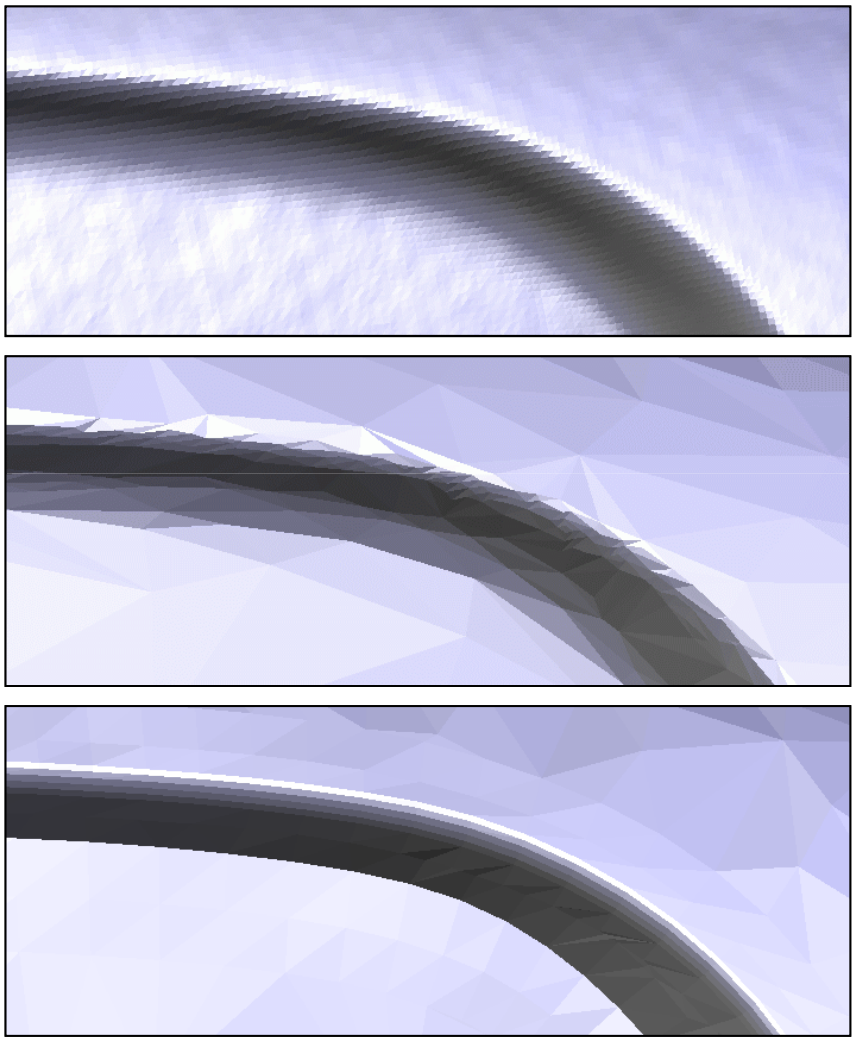
Local Structure

- Element **orientation**



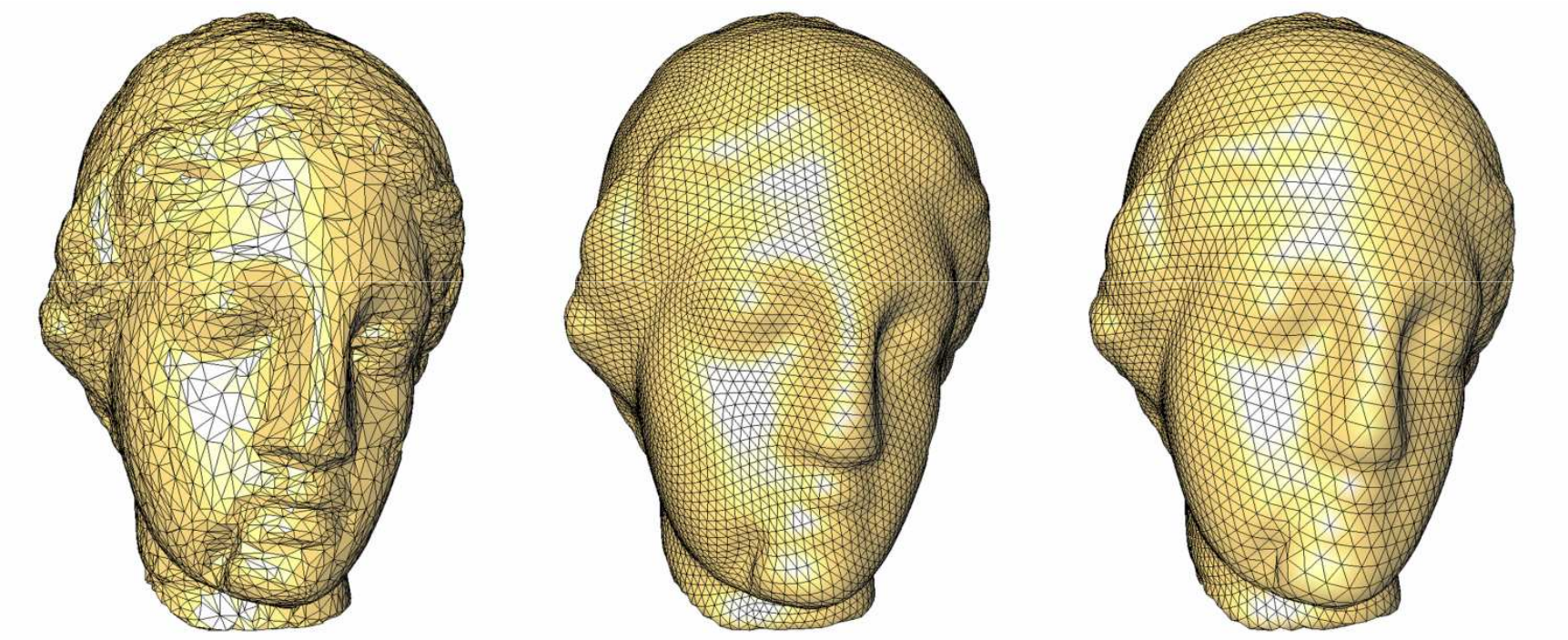
Local Structure

- Element **orientation**



Global Structure

- Mesh **regularity**



Irregular **Semi-regular** **Regular**

Course Notes

Remeshing

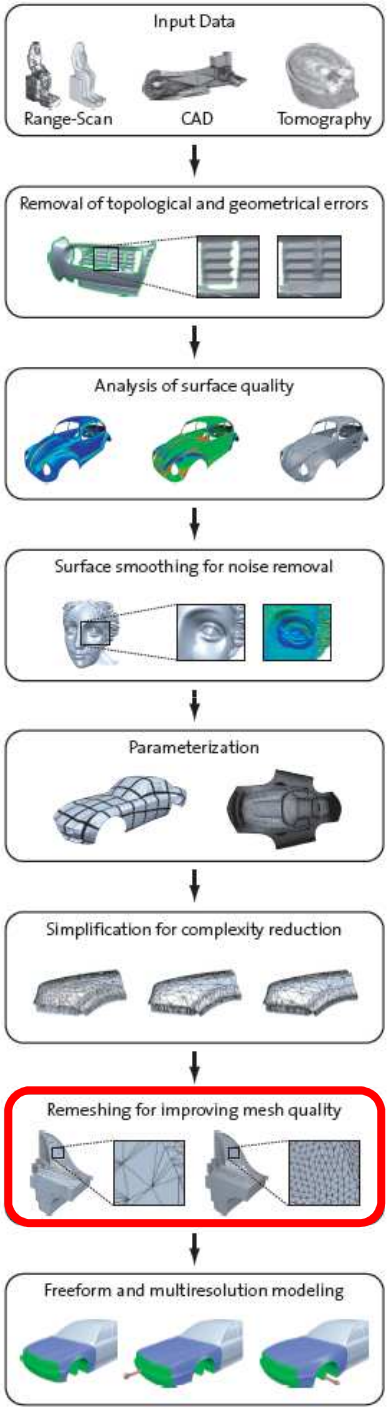
- Isotropic
- Quadrangle
- Error-driven

Today

Remeshing

- Isotropic
- Quadrangle
- Error-driven

} Variable speed, non exhaustive

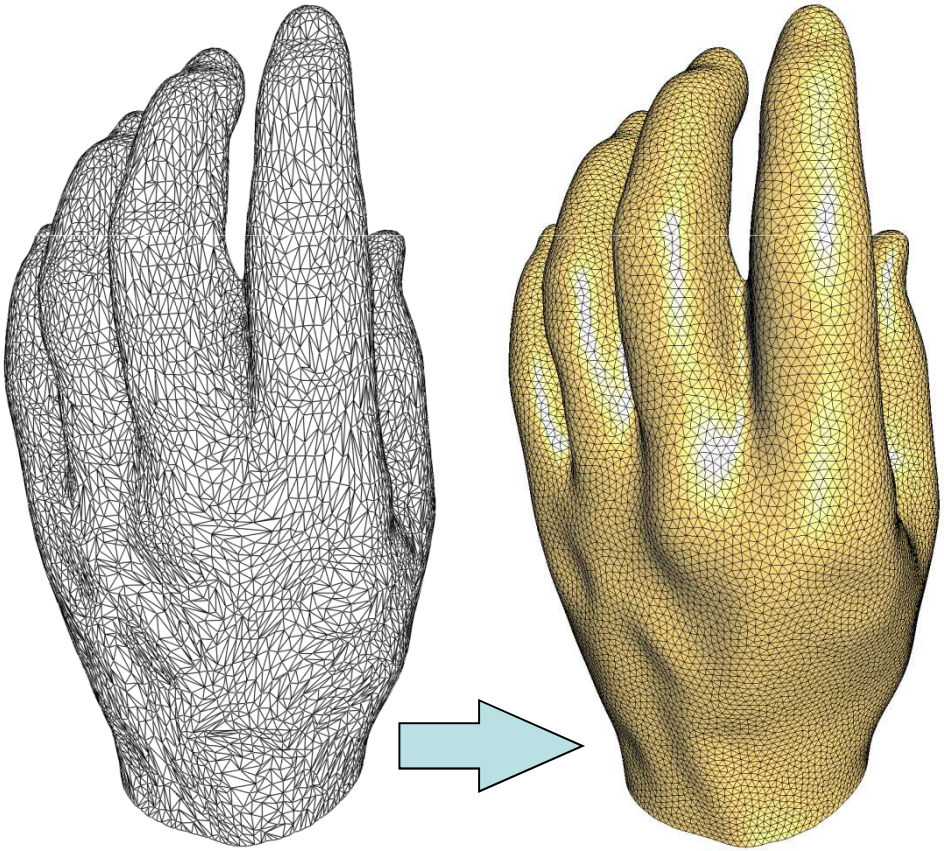


Isotropic Remeshing

Isotropic Remeshing

Well-shaped elements

- for processing & simulation (numerical stability & efficiency)



Main Paradigms

- Greedy
- Variational
- Pliant

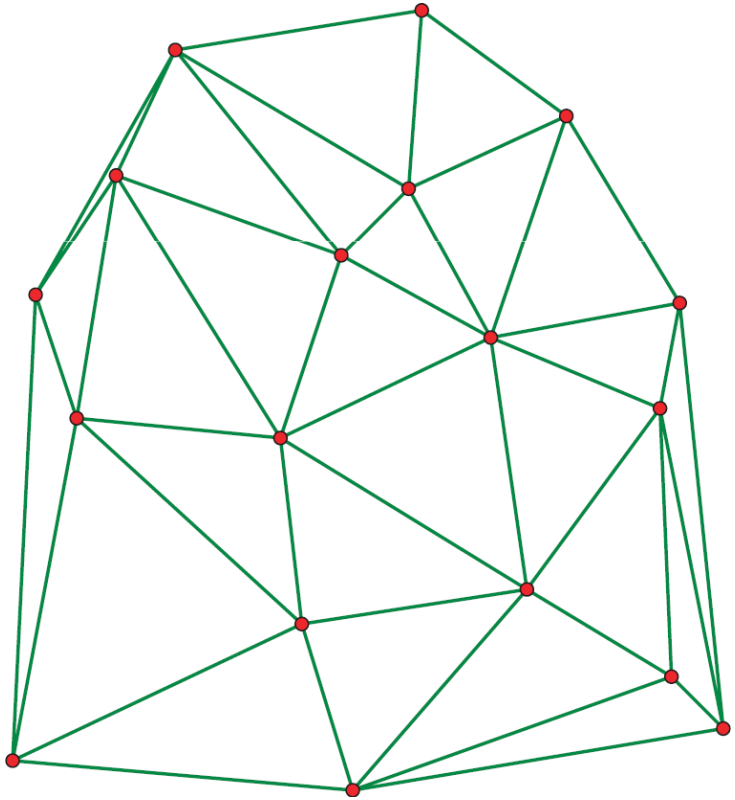
Greedy Surface Remeshing

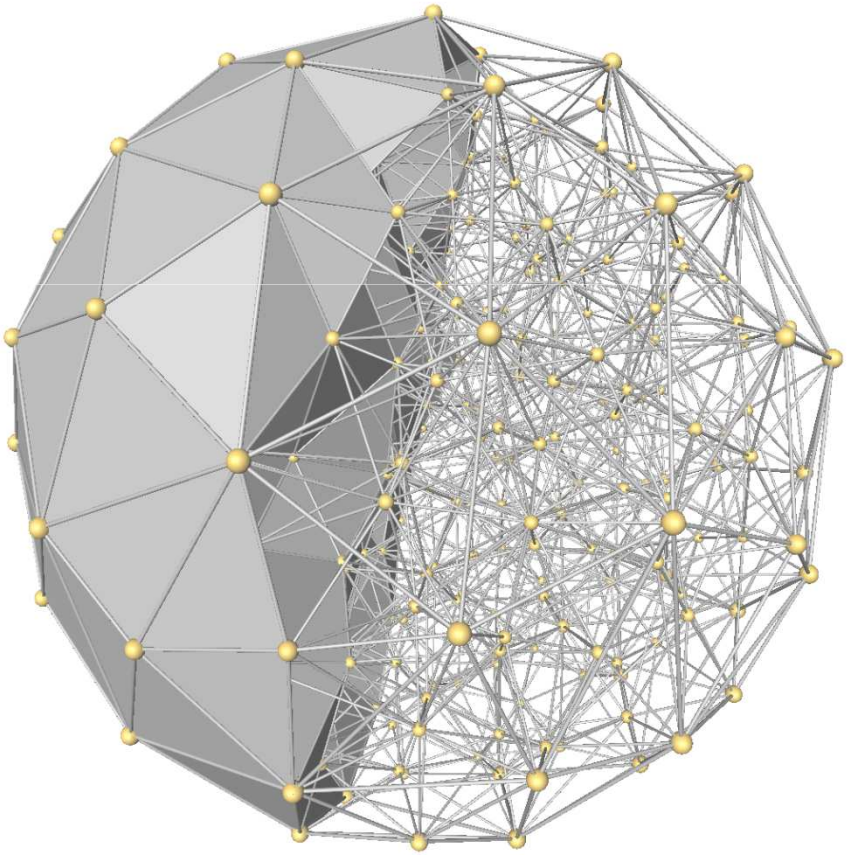
Delaunay refinement and filtering
(a Computational Geometry Approach)

[Chew, Ruppert, Shewchuk, Dey, Boissonnat-Oudot]

Delaunay Triangulation

- canonical triangulation associated to any point set

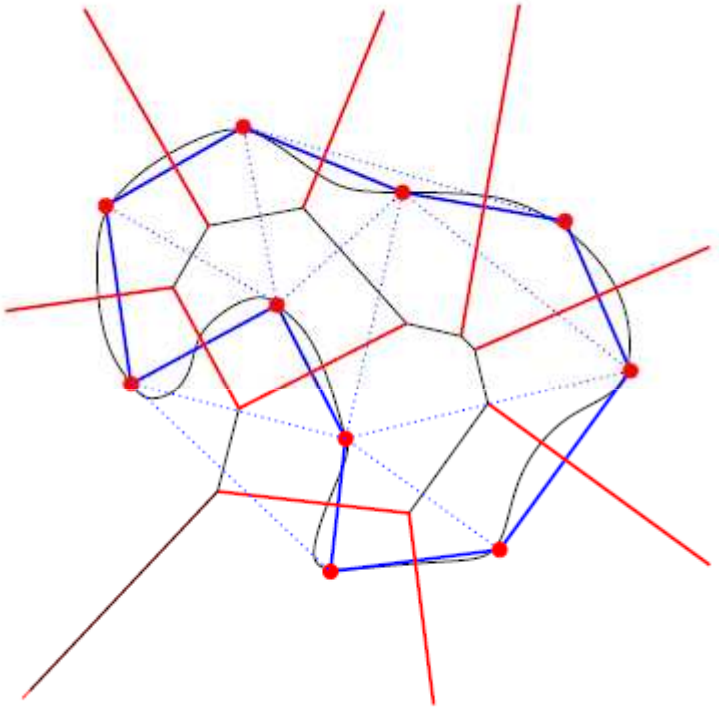




Delaunay Filtering

Definition

The restricted Delaunay triangulation $\mathrm{Del}_{|S}(E)$ is the set of facets of the Delaunay triangulation whose dual edges intersect the surface

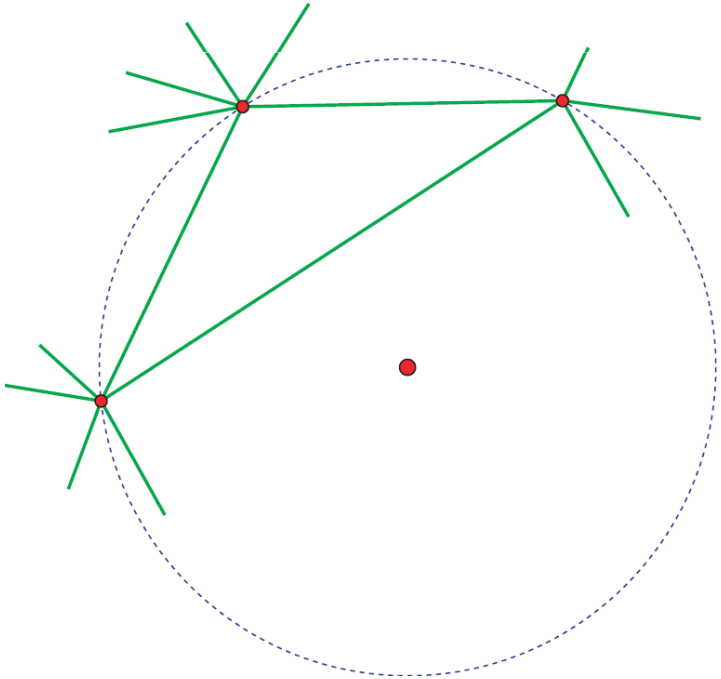


(2D)

Delaunay Refinement

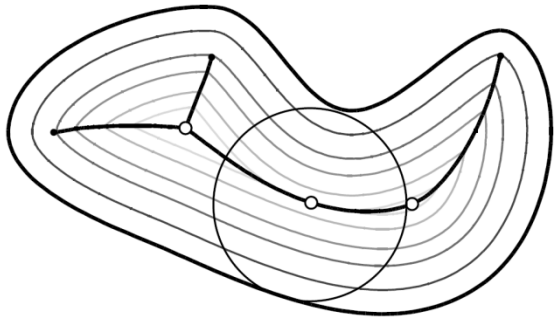
- **key idea**: break bad elements by inserting circumcenters (Voronoi vertices) [Chew, Ruppert, Shewchuk,...]

  “bad” in terms of size or shape



Surface Remeshing

$0 < \varepsilon < \mathrm{rch}(S)$

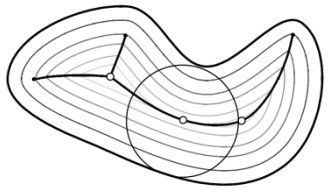


ORACLE : For a facet f of $\mathrm{Del}_{|S}(E)$, return c_f and r_f

A facet f is bad if $r_f > \varepsilon$

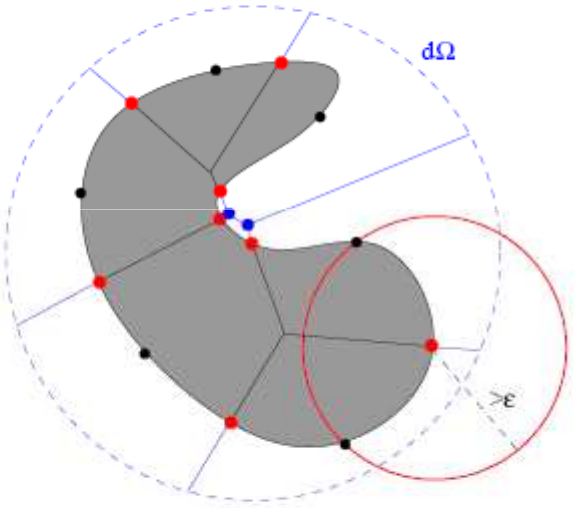
Surface Remeshing

$0 < \varepsilon < \mathrm{rch}(S)$



ORACLE : For a facet f of $\mathrm{Del}_{|S}(E)$,
return c_f and r_f

A facet f is bad if $r_f > \varepsilon$



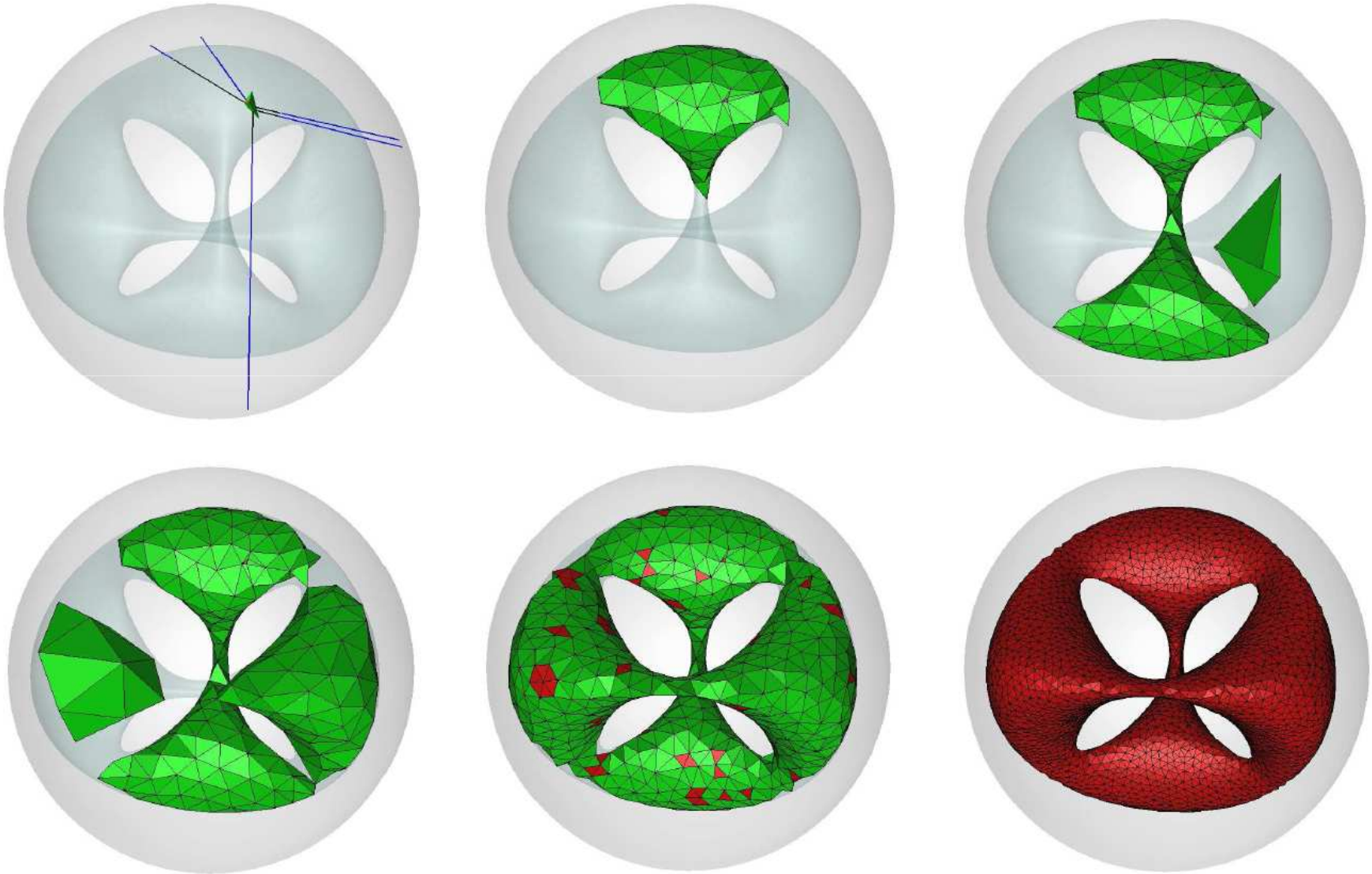
Algorithm

INIT compute an initial (small) sample $E_0 \subset S$

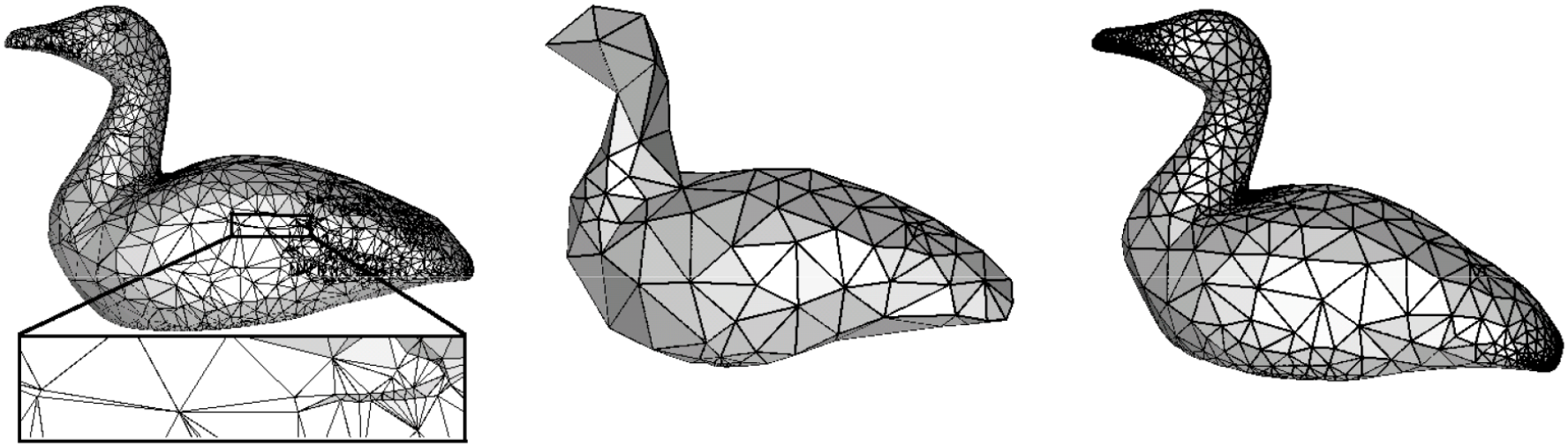
REPEAT pick a bad facet f
insert c_f in $\mathrm{Del}(E)$
update E and $\mathrm{Del}_{|S}(E)$

UNTIL all facets are good

Surface Remeshing at Work

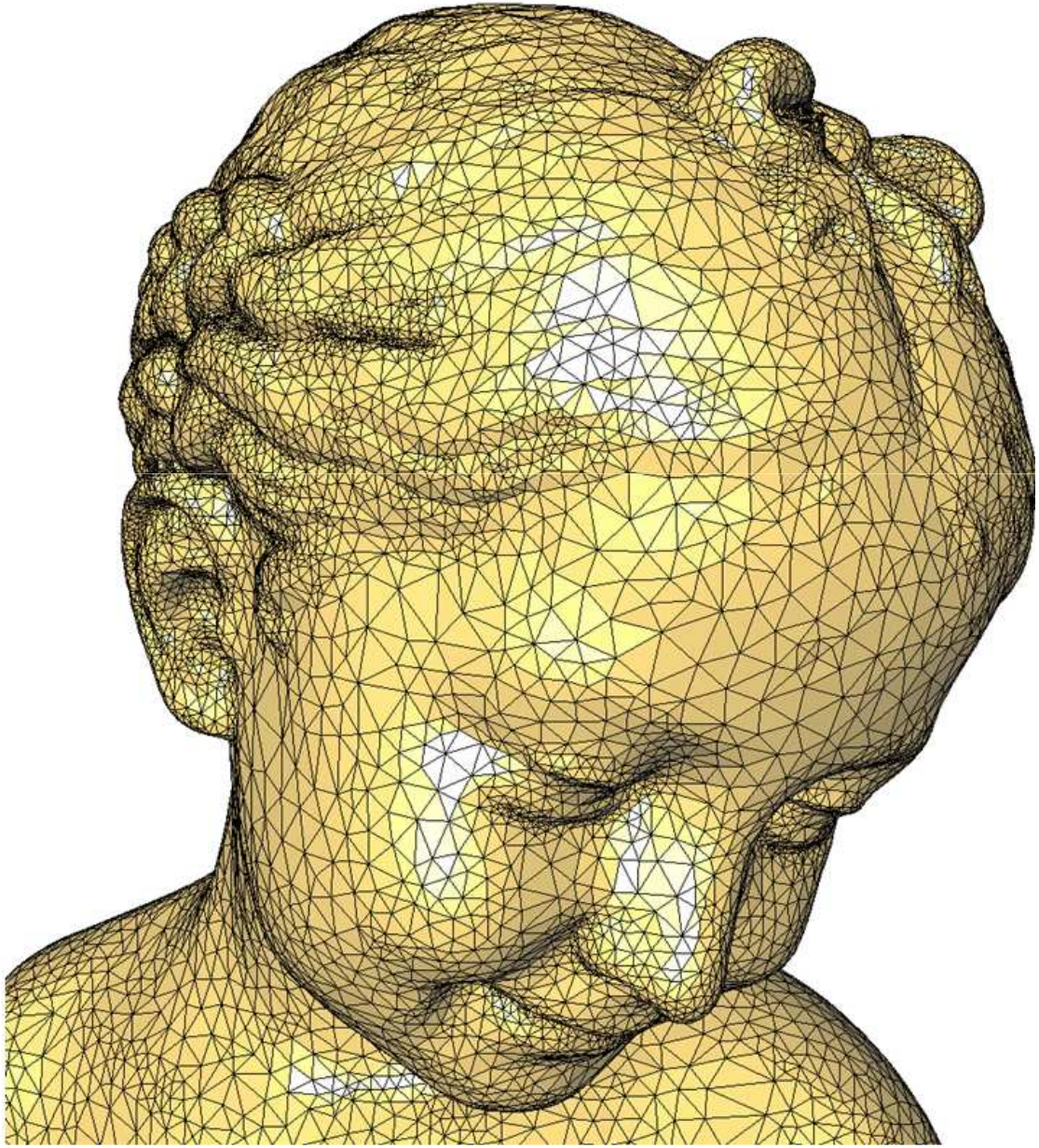


Example: Remeshing

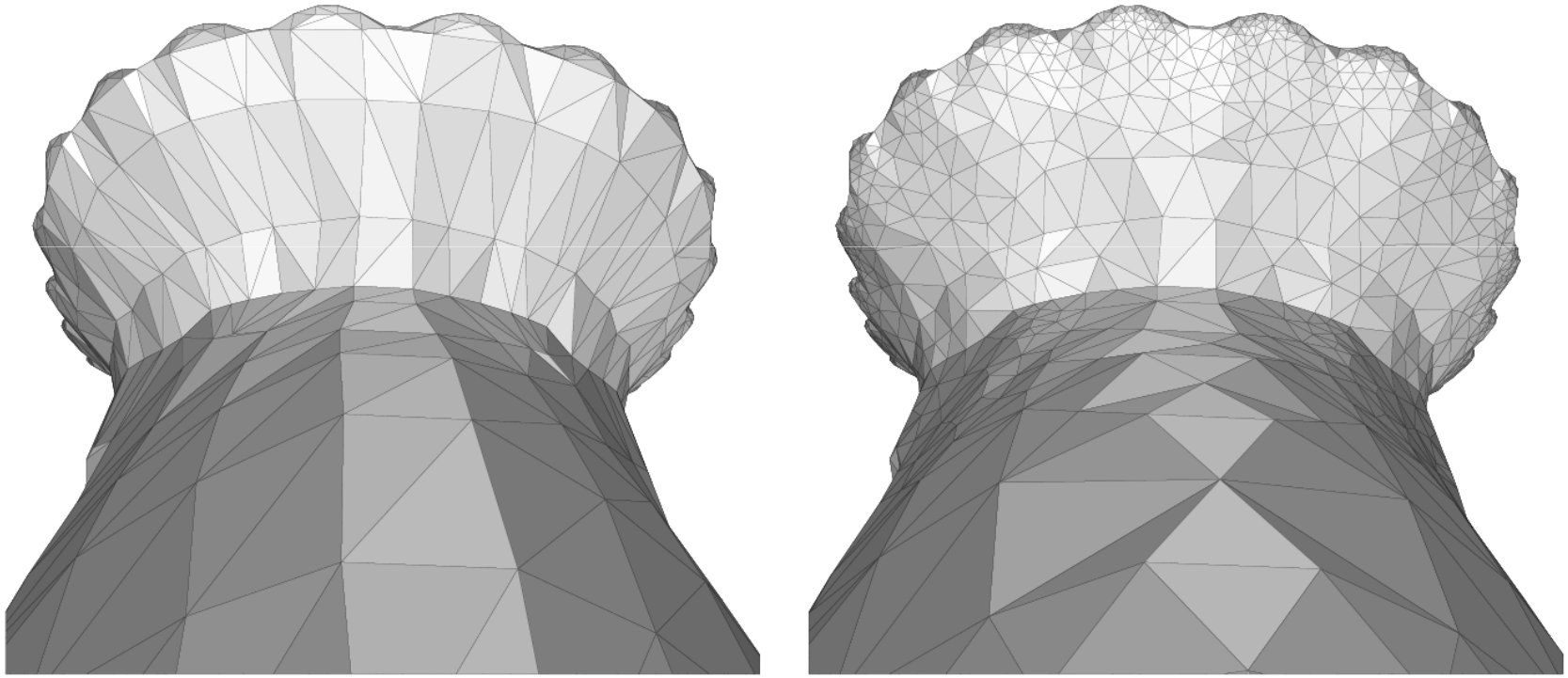


[Dey-Li-Ray] IMRT 05, *Polygonal surface remeshing with Delaunay refinement*

Example: Remeshing



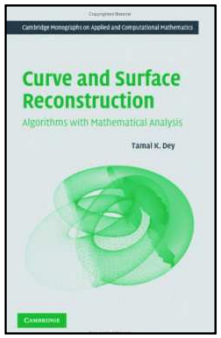
Example: Removing Skinny Triangles



[Boissonnat-Oudot 03]

Guarantees

- Produces a good approximation of the surface
  - $\hat{S}$ is isotopic to S
  - $d_H(\hat{S}, S) = O_S(\varepsilon^2)$, error on normals, area $= O(\varepsilon)$
  - S is covered by the surface Delaunay balls of $\hat{S}$
- Produces sparse samples of optimal size
  - the set E of vertices of $\hat{S}$ is a sparse 2ε-sample of S
  - $|E| = O(\frac{\text{area}(S)}{\varepsilon^2})$
- The aspect ratio of the facets can be controlled



[T.K.Dey] *Curve and Surface Reconstruction : Algorithms with Mathematical Analysis*

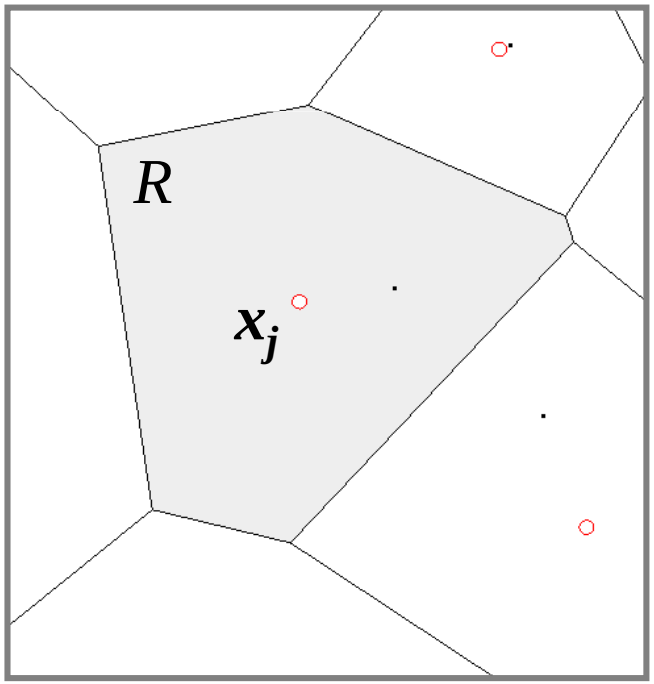
Variational

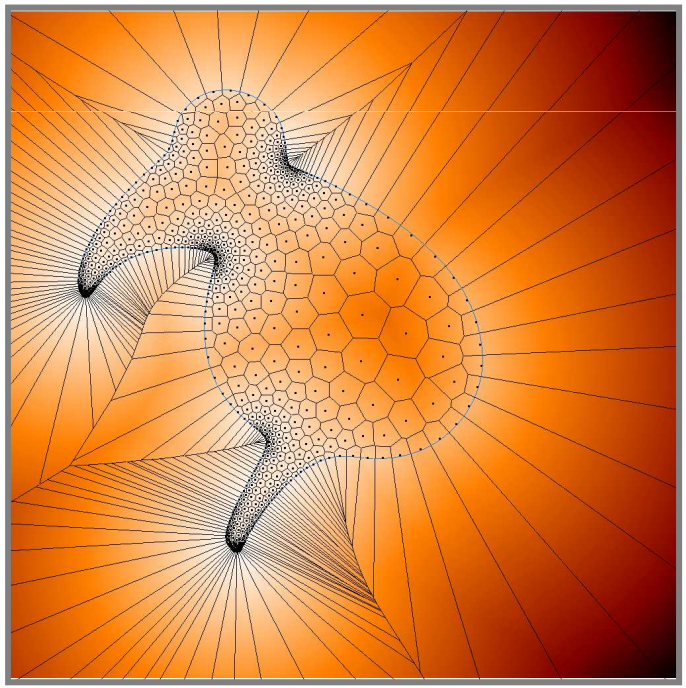
Proposal:

- design one energy function
- good solutions correspond to low-energy ones
- solutions found by optimization techniques

Energy

$$E = \sum_{j=1..k} \int_{x \in R_j} \rho(x) \| x - x_j \|^2 \, dx$$

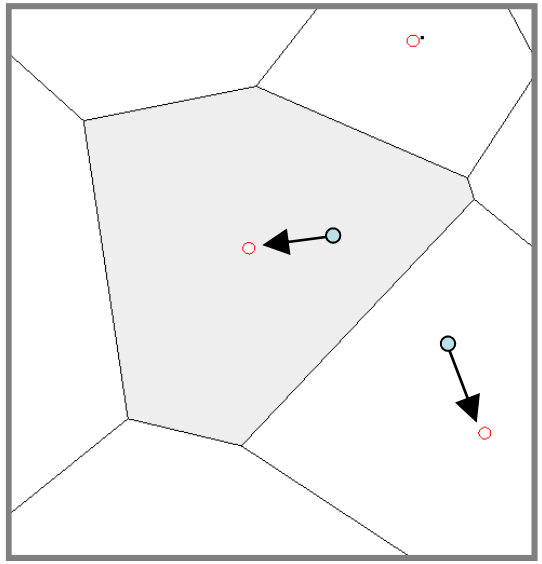




Lloyd Algorithm

- **Alternate:**
  - Voronoi partitioning
  - move sites to respective centroids
- **Minimizes energy**
  - necessary condition for optimality: Centroidal Voronoi tessellation

demo



Isotropic Remeshing

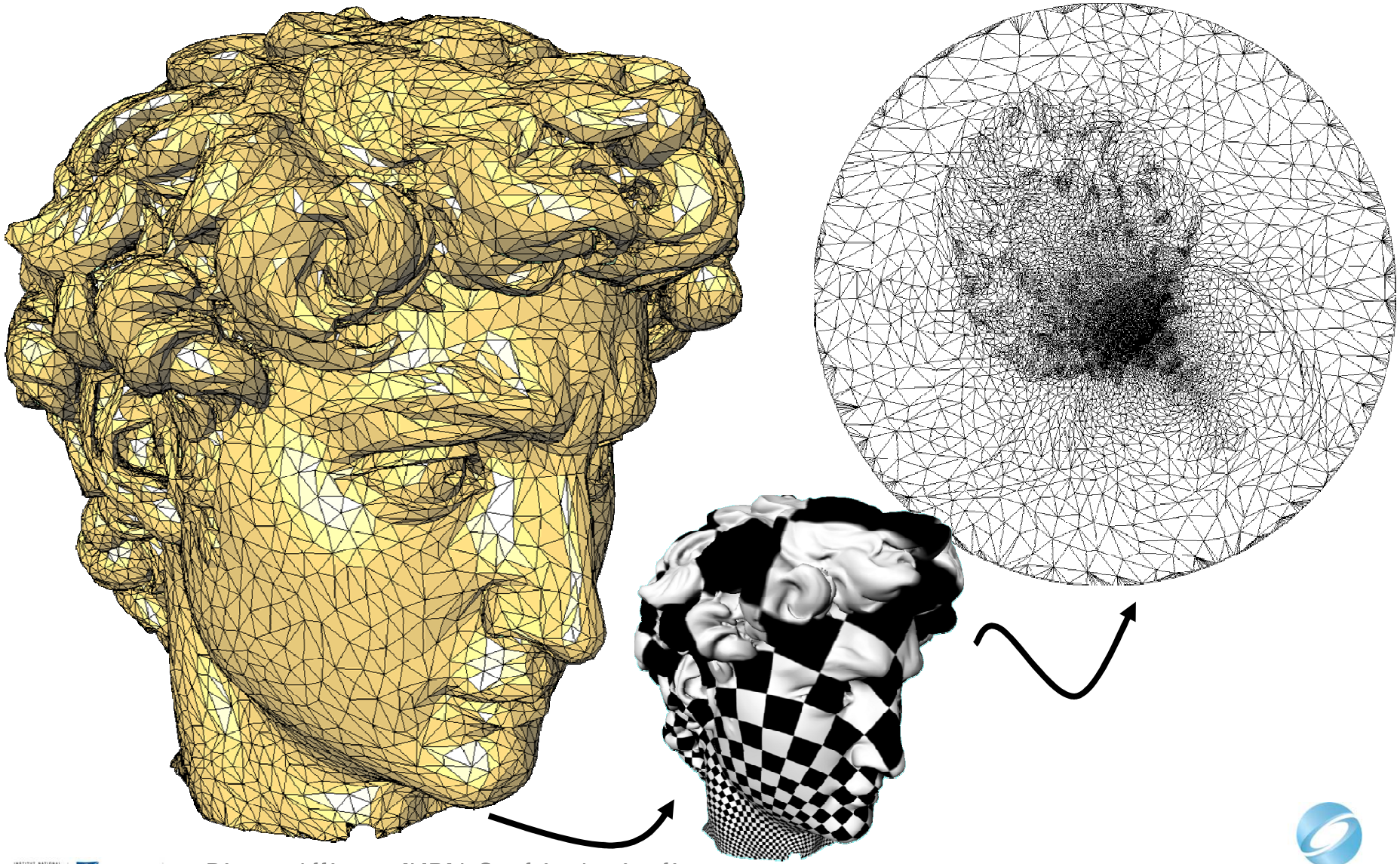
Global parameterization:

[A., Colin de Verdiere, Devillers and Isenburg] SMI 03, *Isotropic Surface Remeshing*.

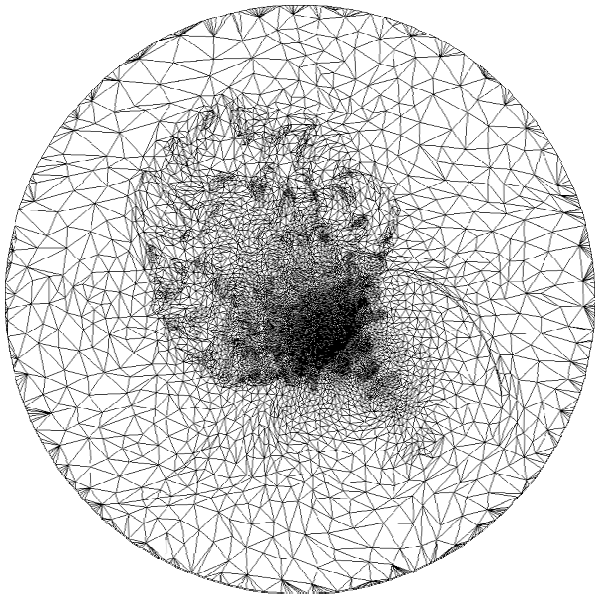
Local parameterization:

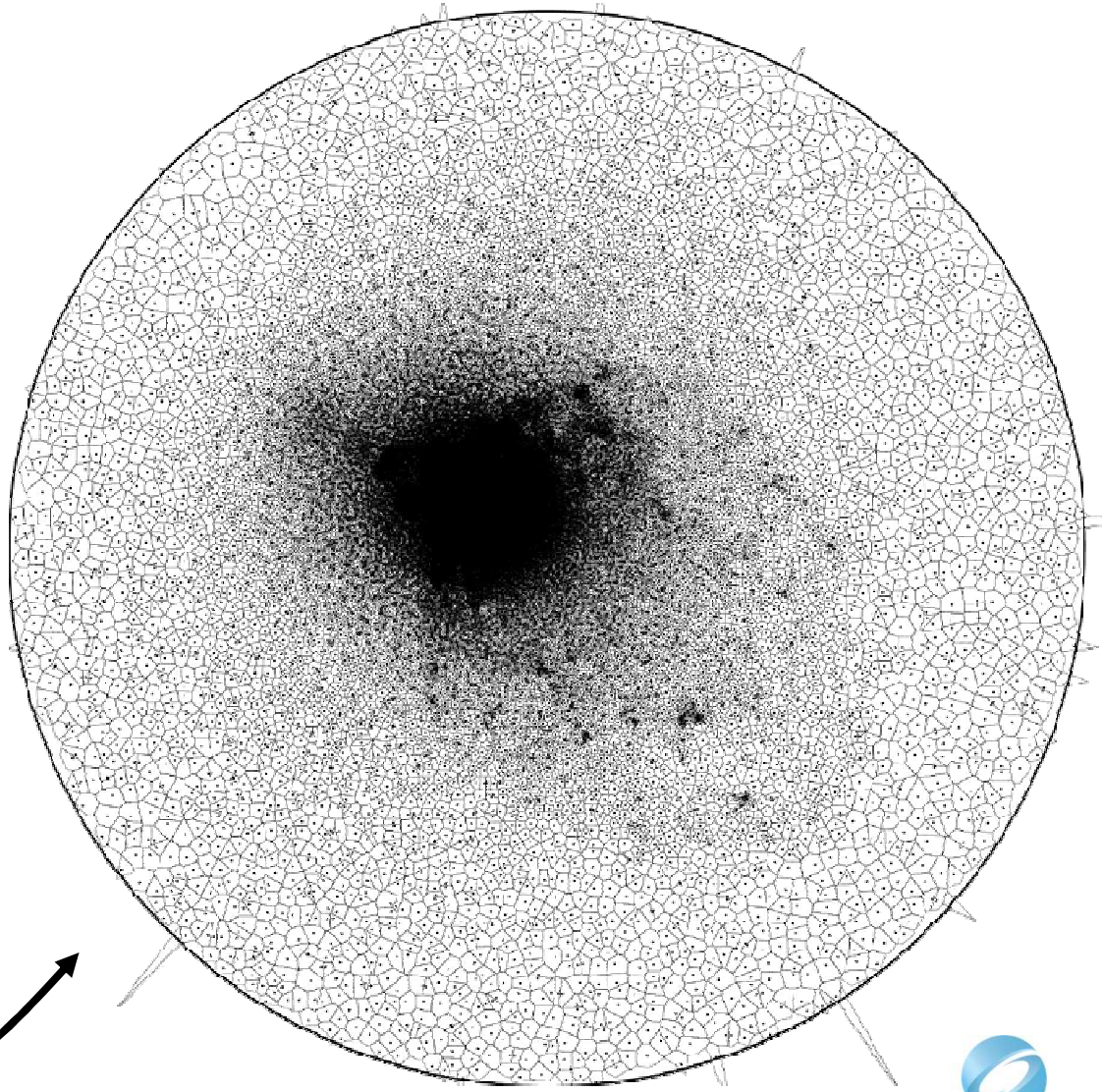
[Surazhsky, A., Gotsman] IMRT 03, *Isotropic Remeshing of Surfaces: a Local Parameterization Approach*.

1. Global Parameterization (Bruno)



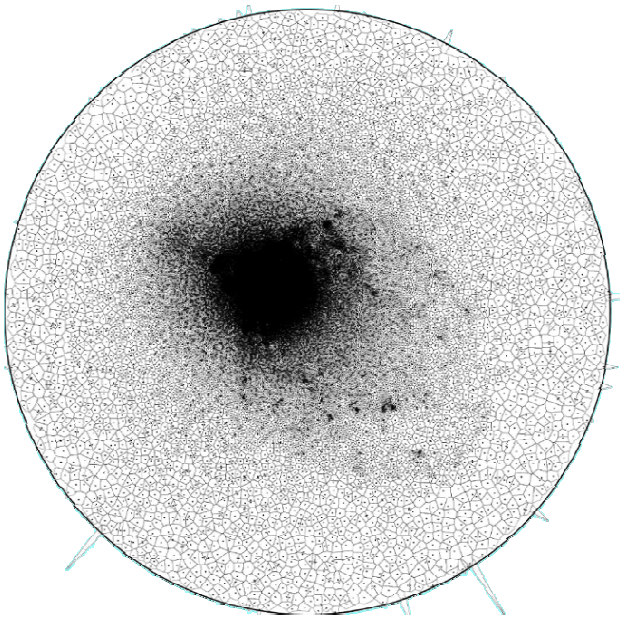
2. Initial Sample Repartition



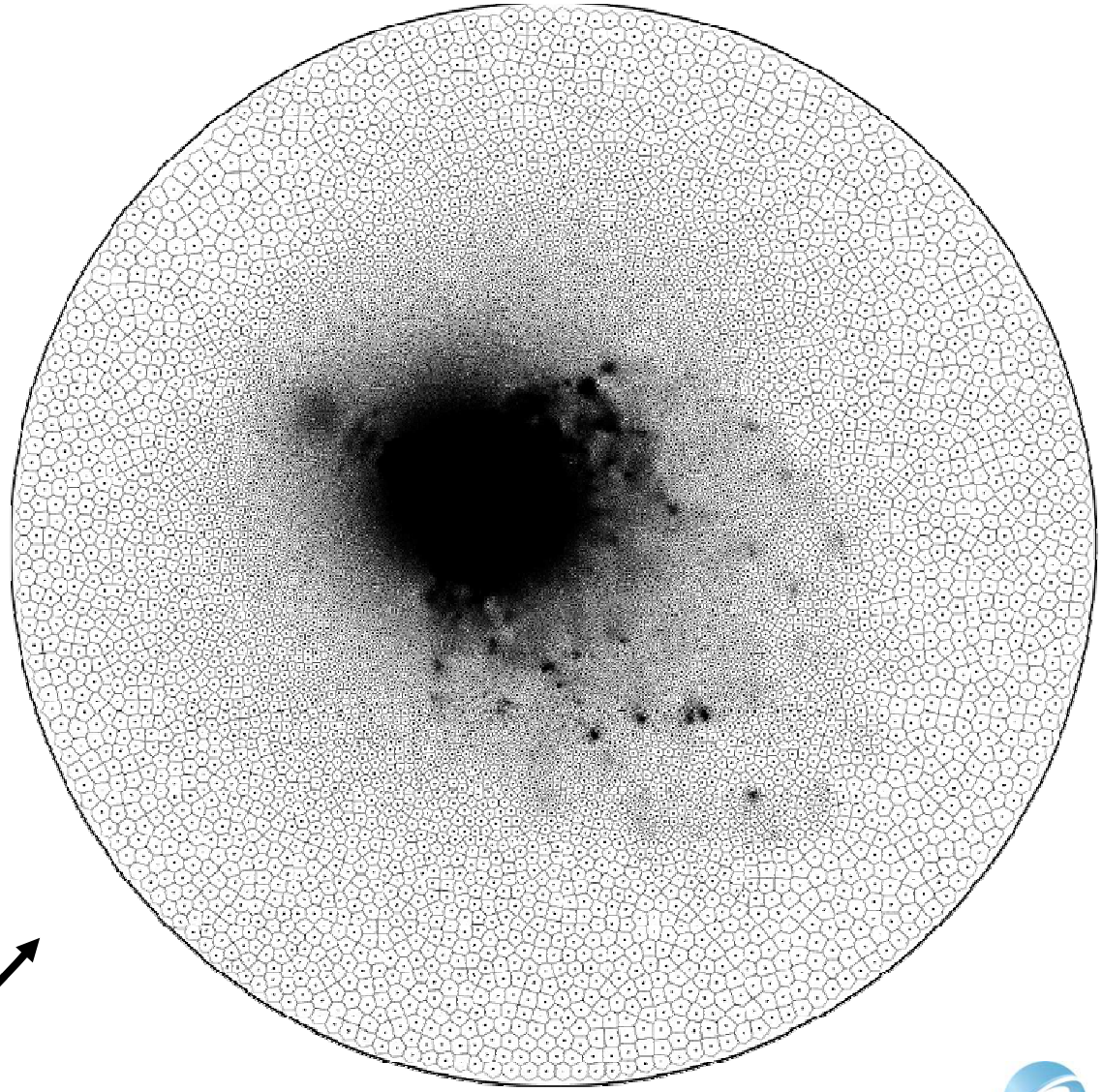


Voronoi diagram

3. Optimized Sample Placement

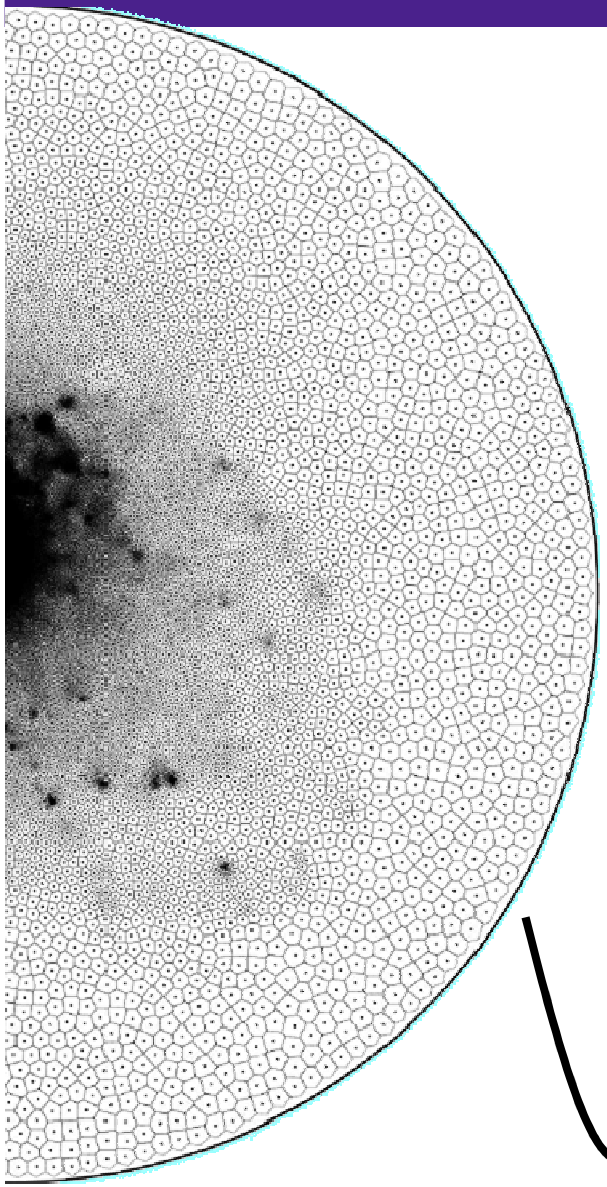


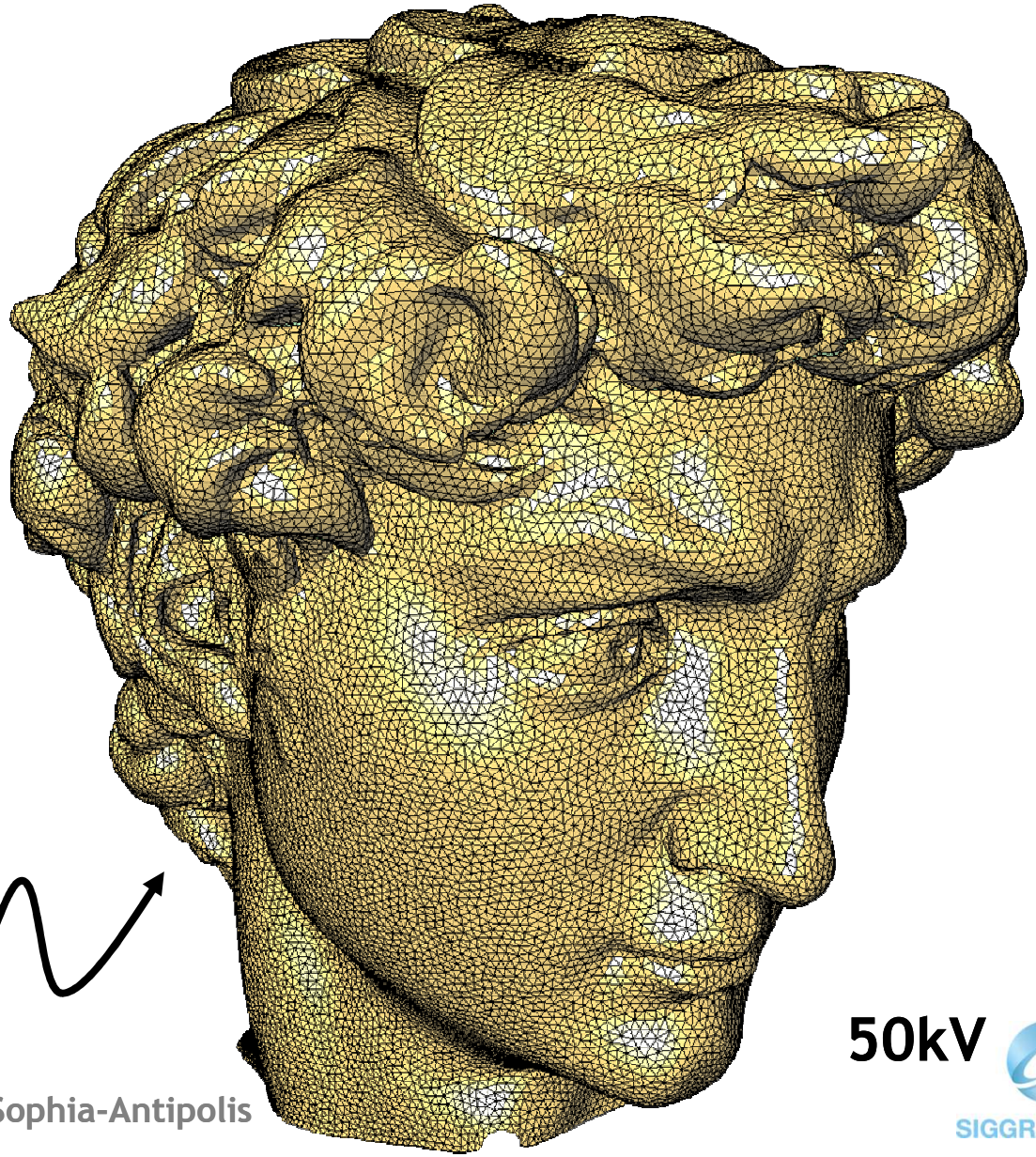
Lloyd



centroidal Voronoi diagram

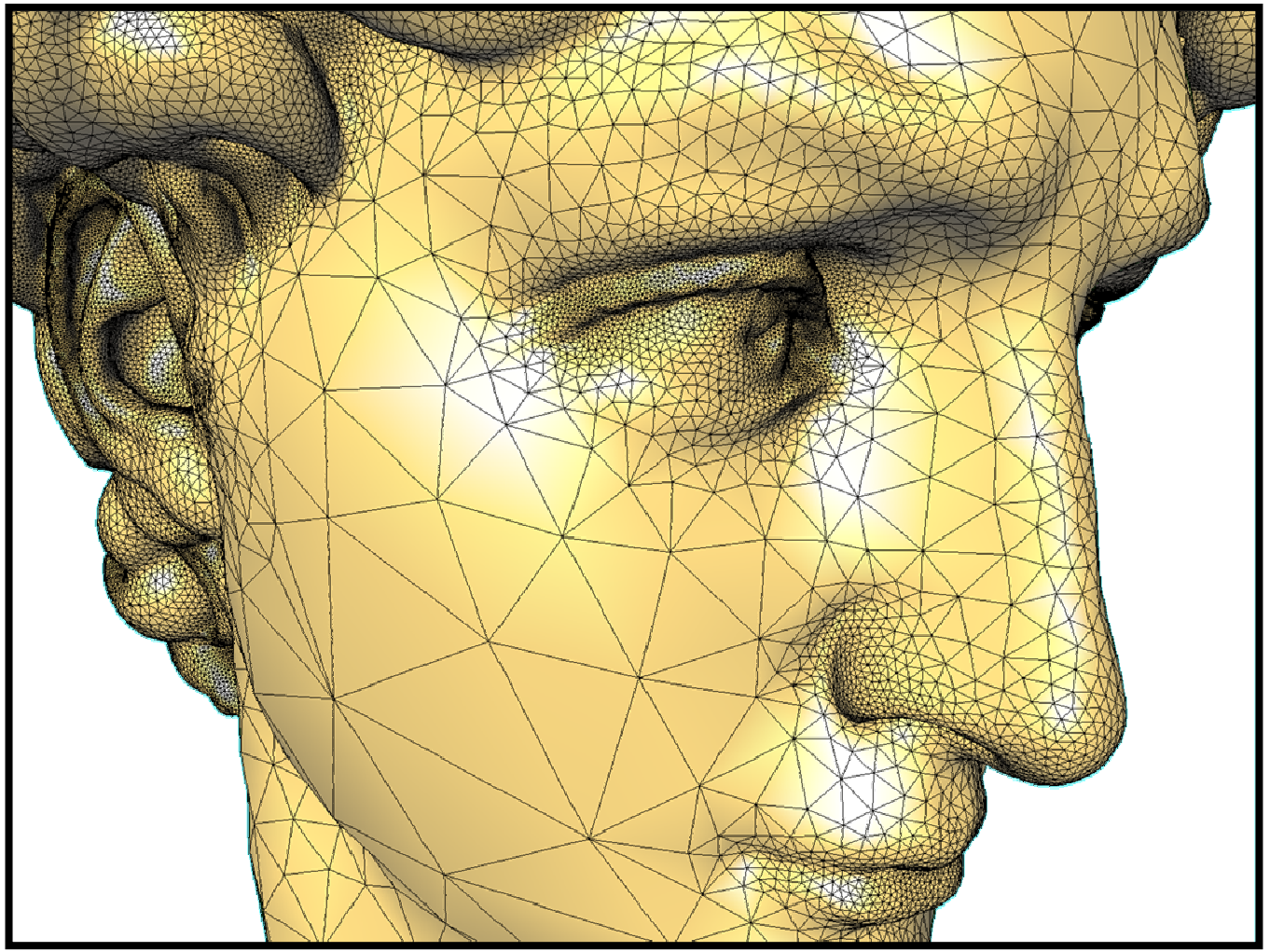
Uniform Remeshing

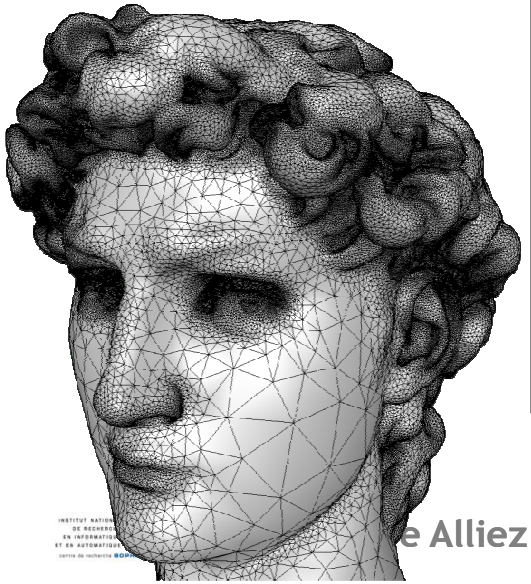




50kV

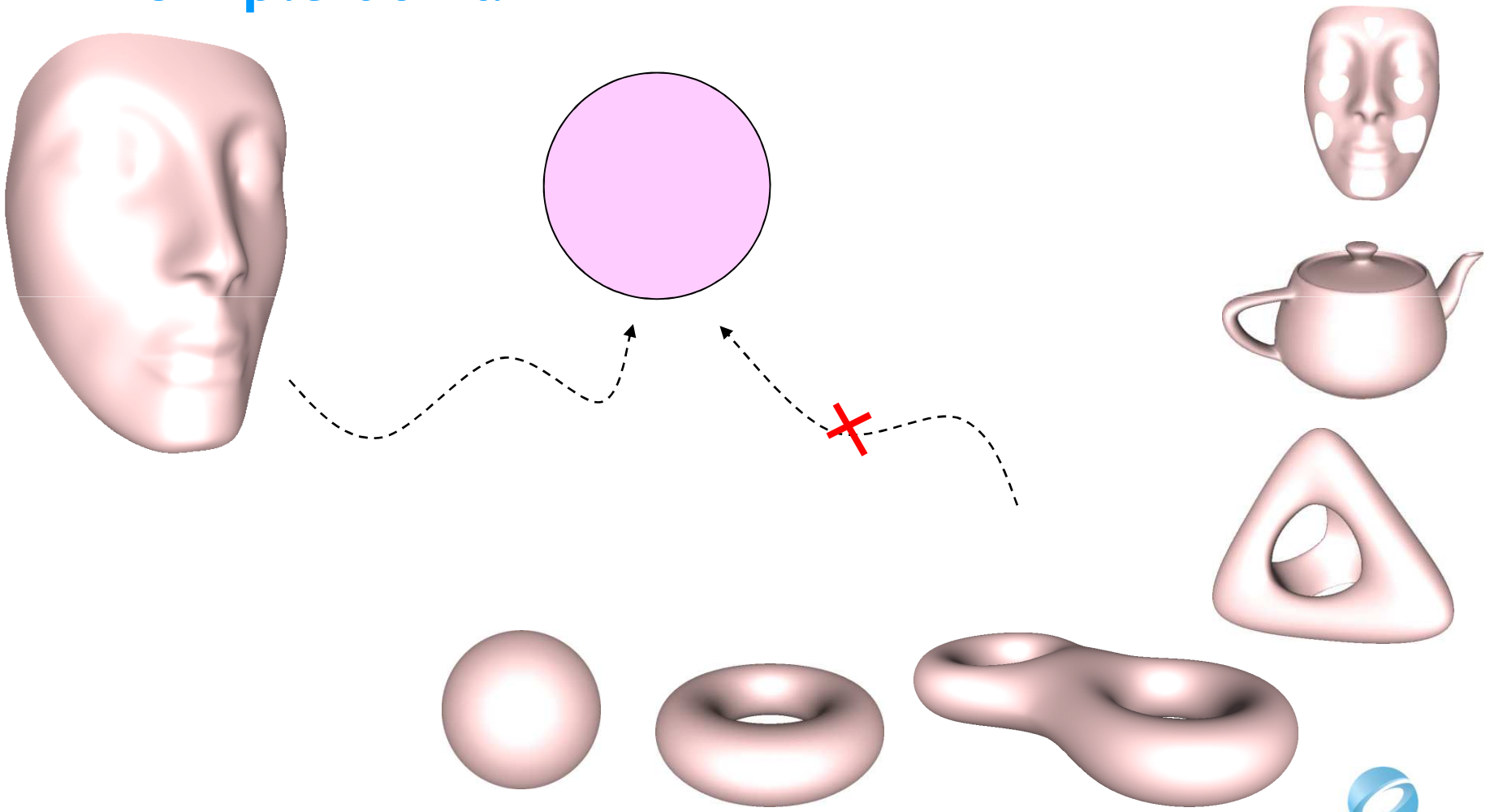
Non-uniform Remeshing



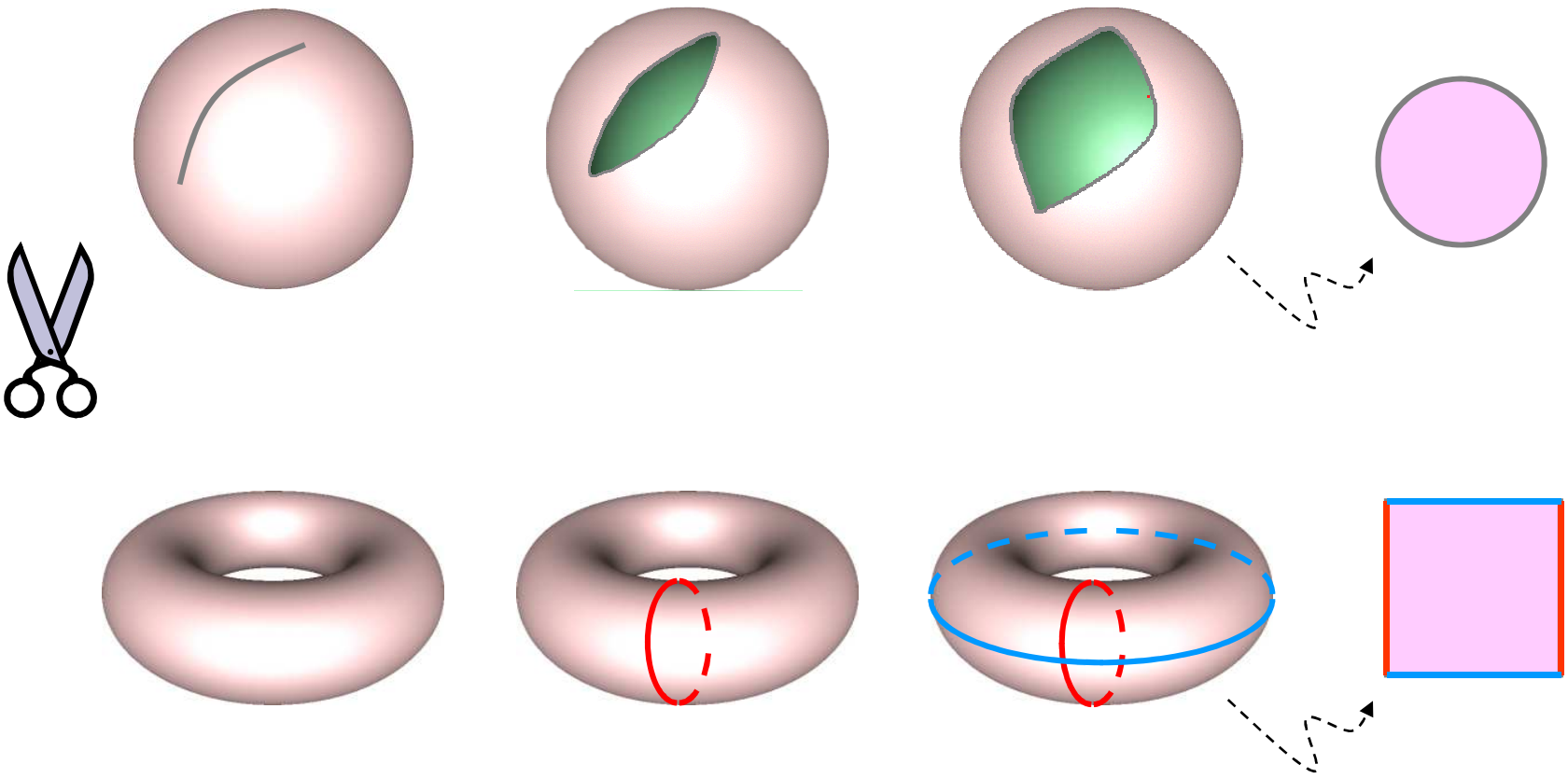


Parameterization

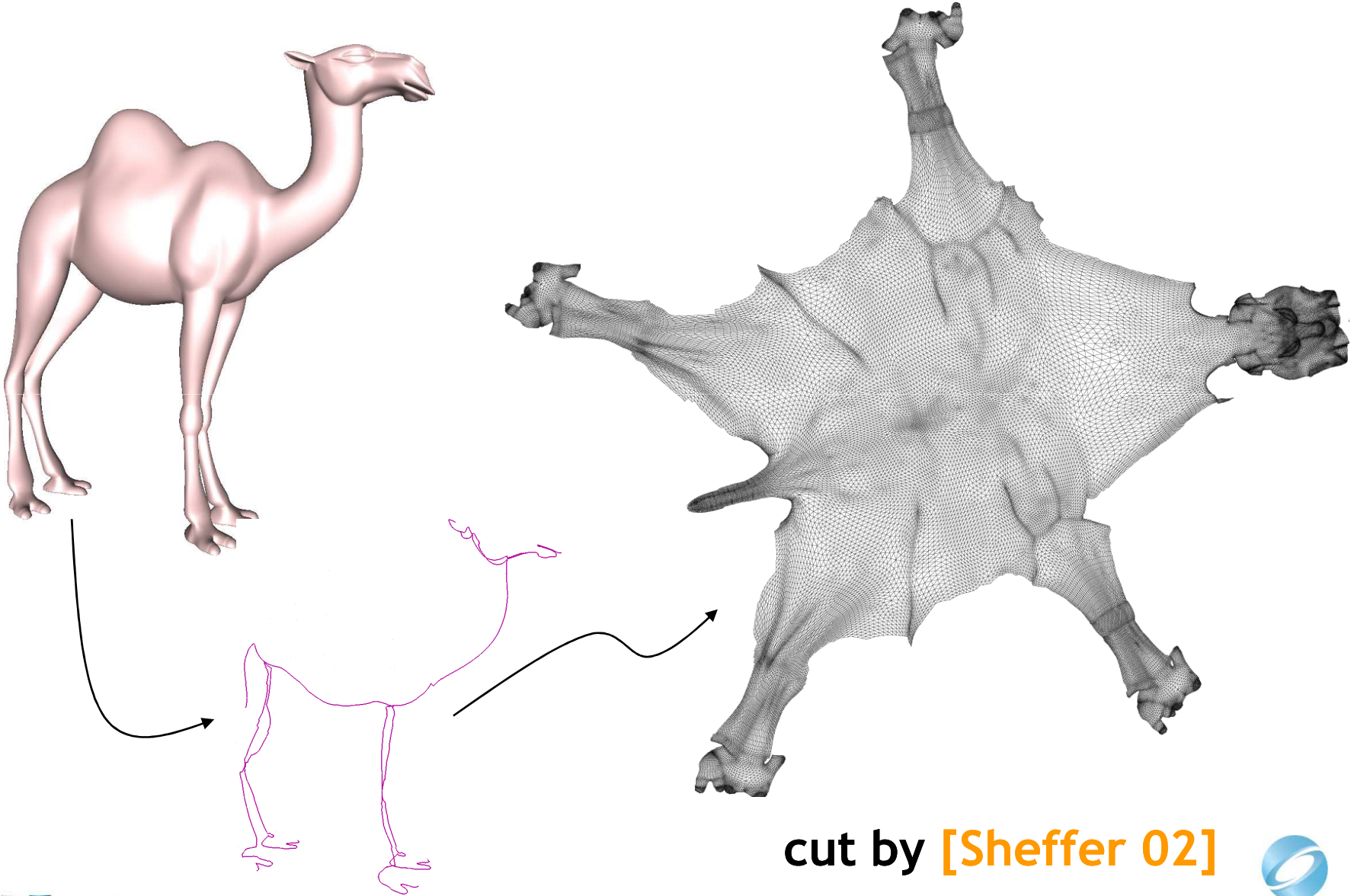
-> simple domain



Surface cutting

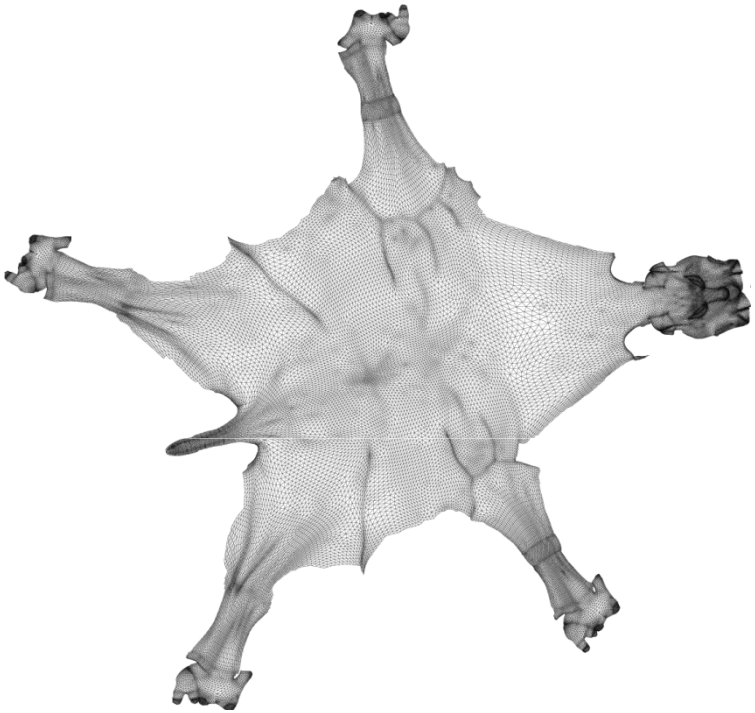


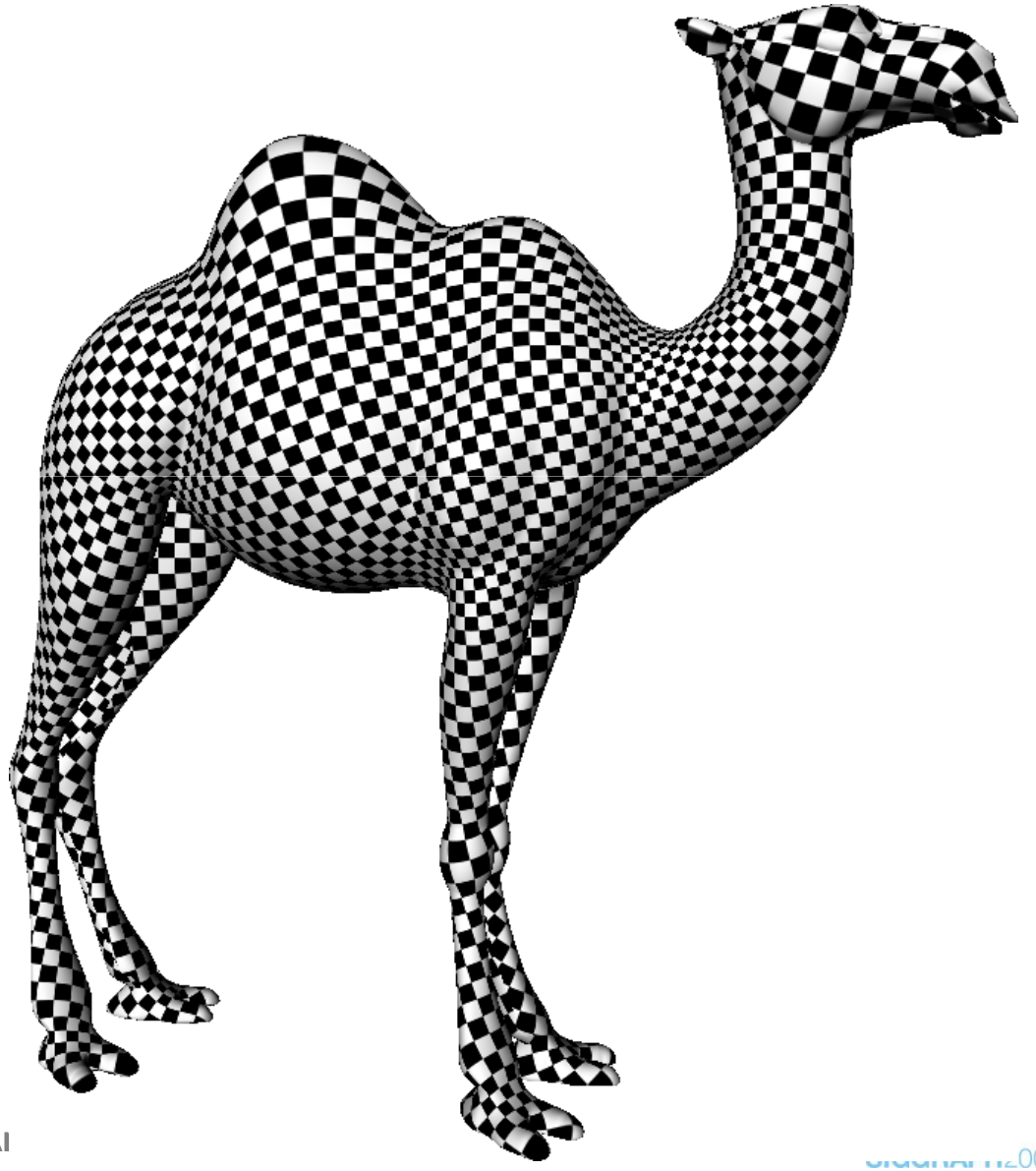
Pelting a Camel



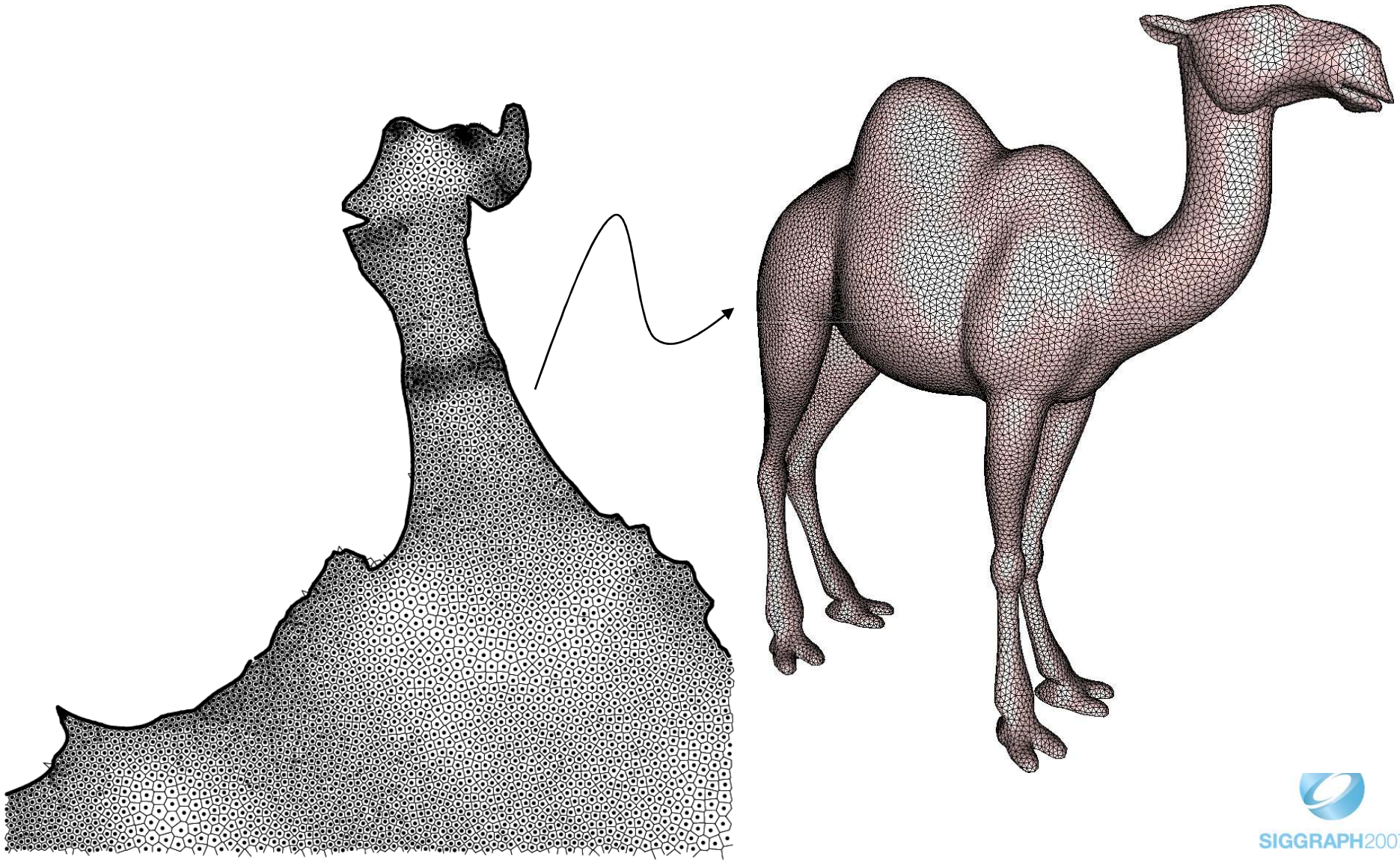
cut by [Sheffer 02]

Parameterization

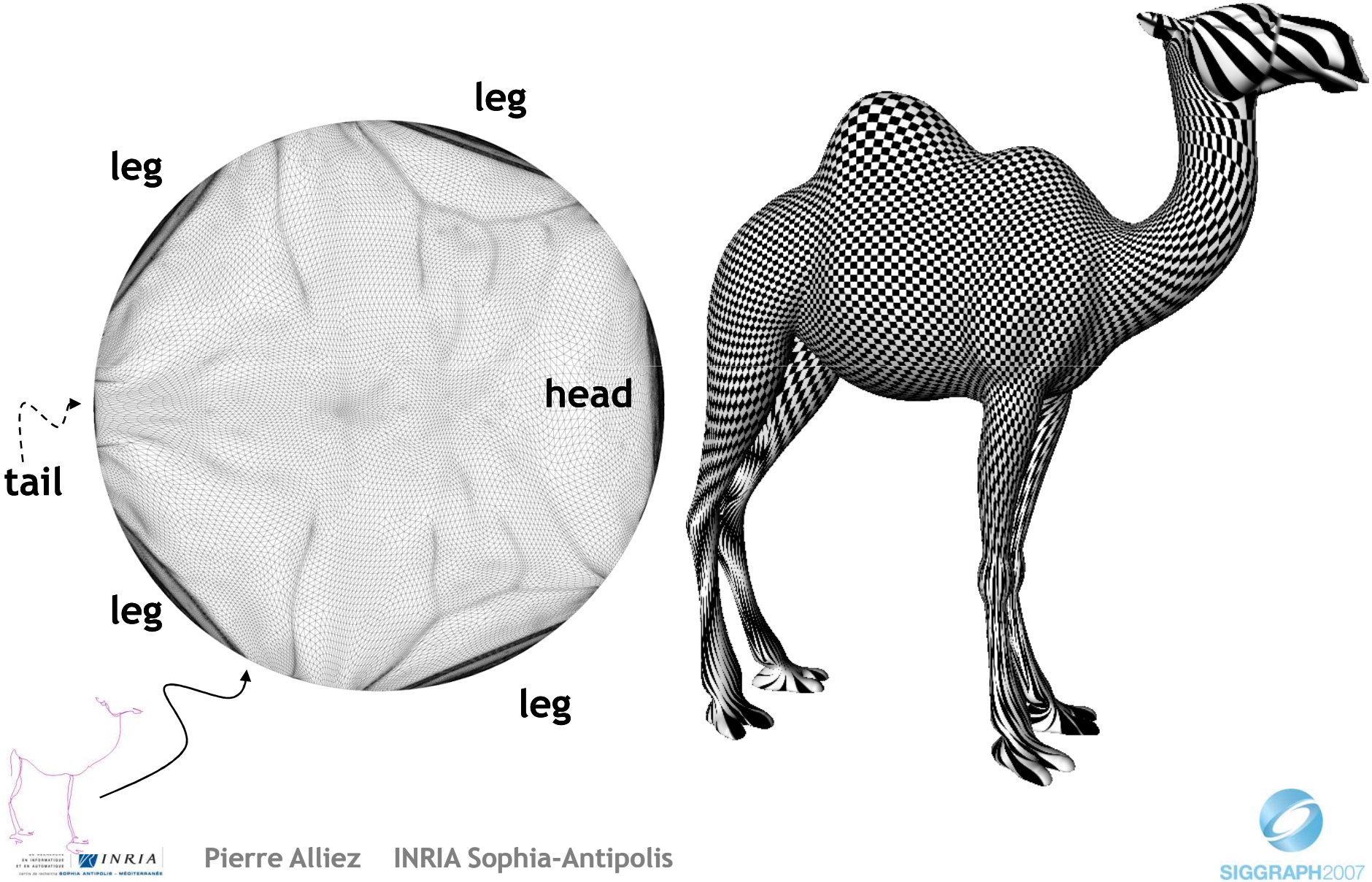




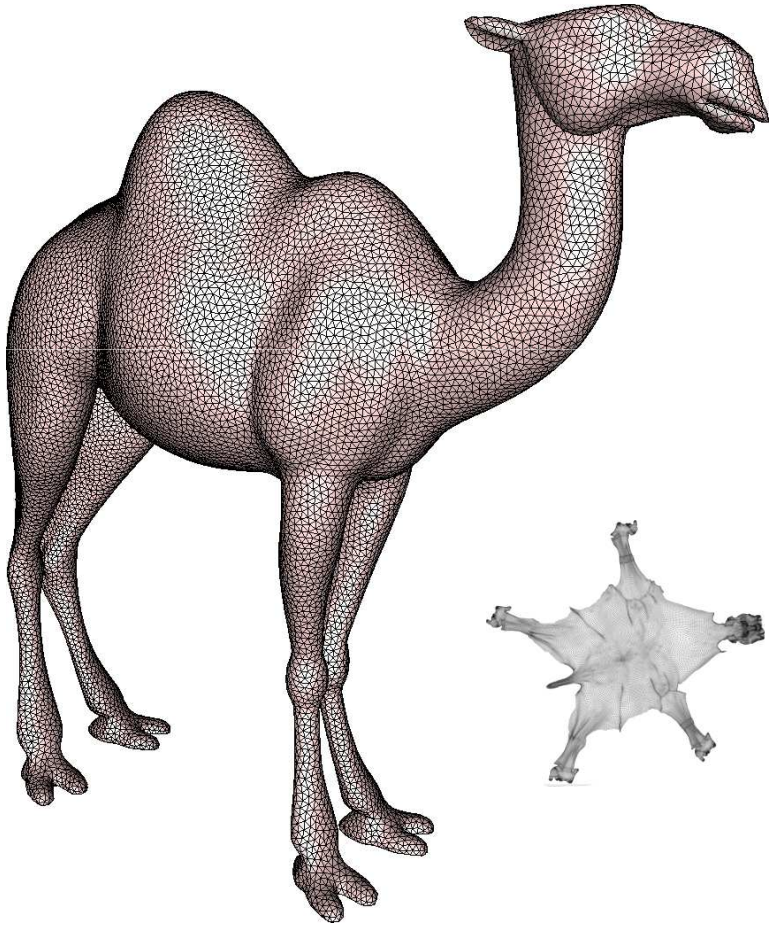
Uniform remeshing

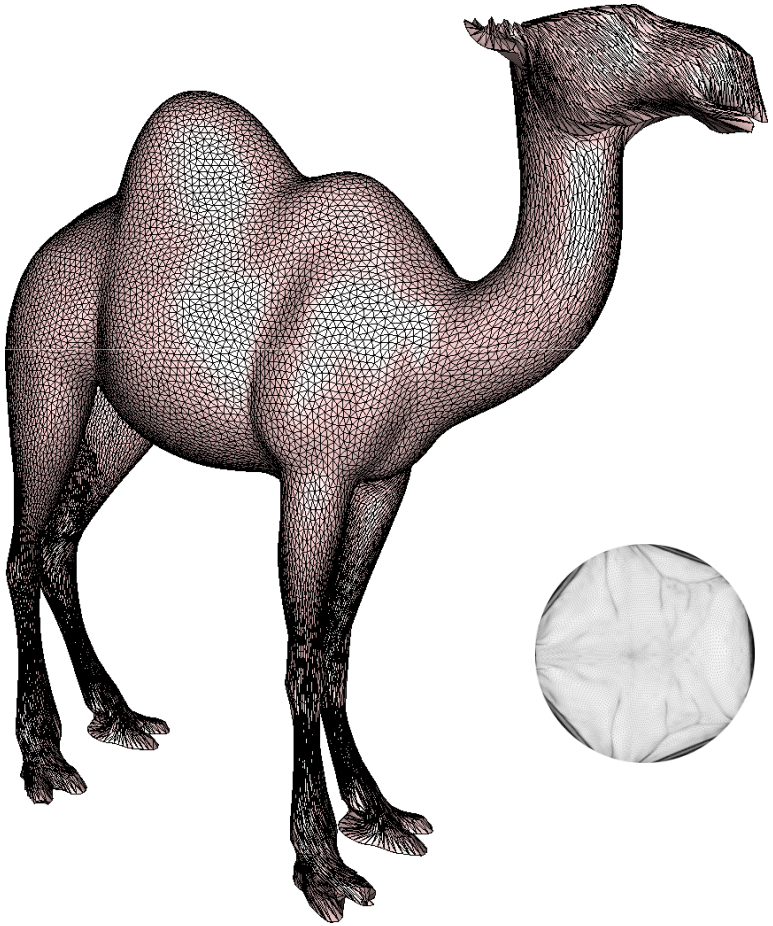


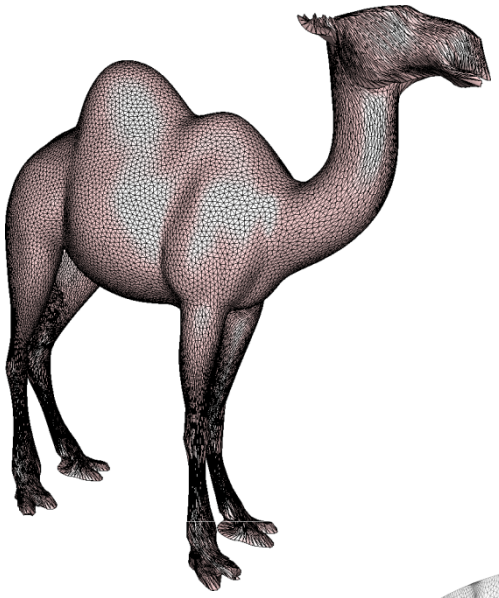
Same cut, fixed boundary ?

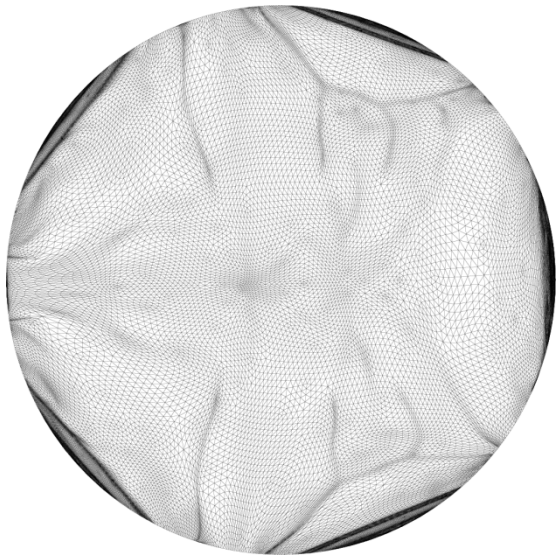


Free vs Fixed Boundary



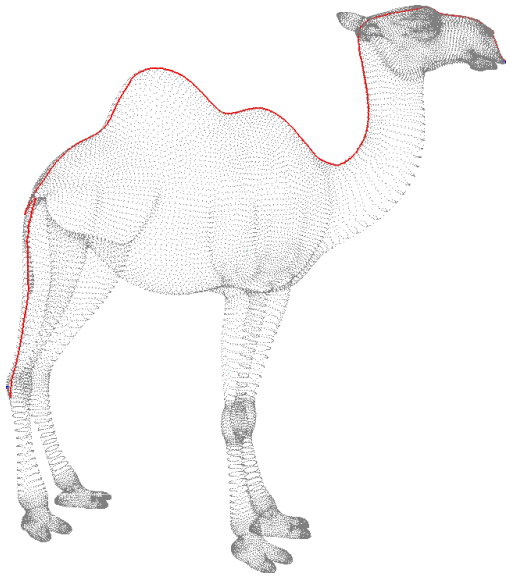


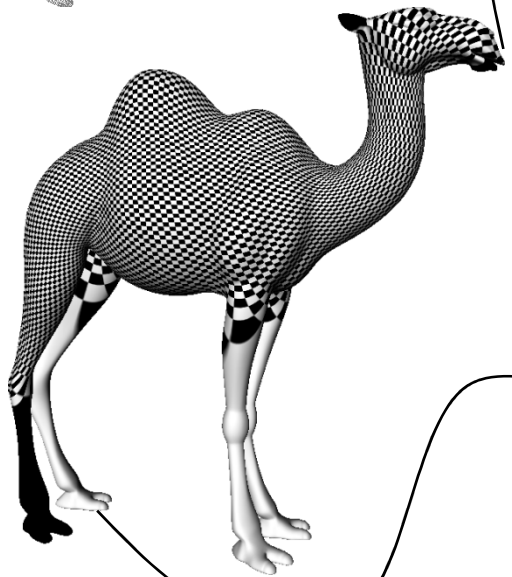


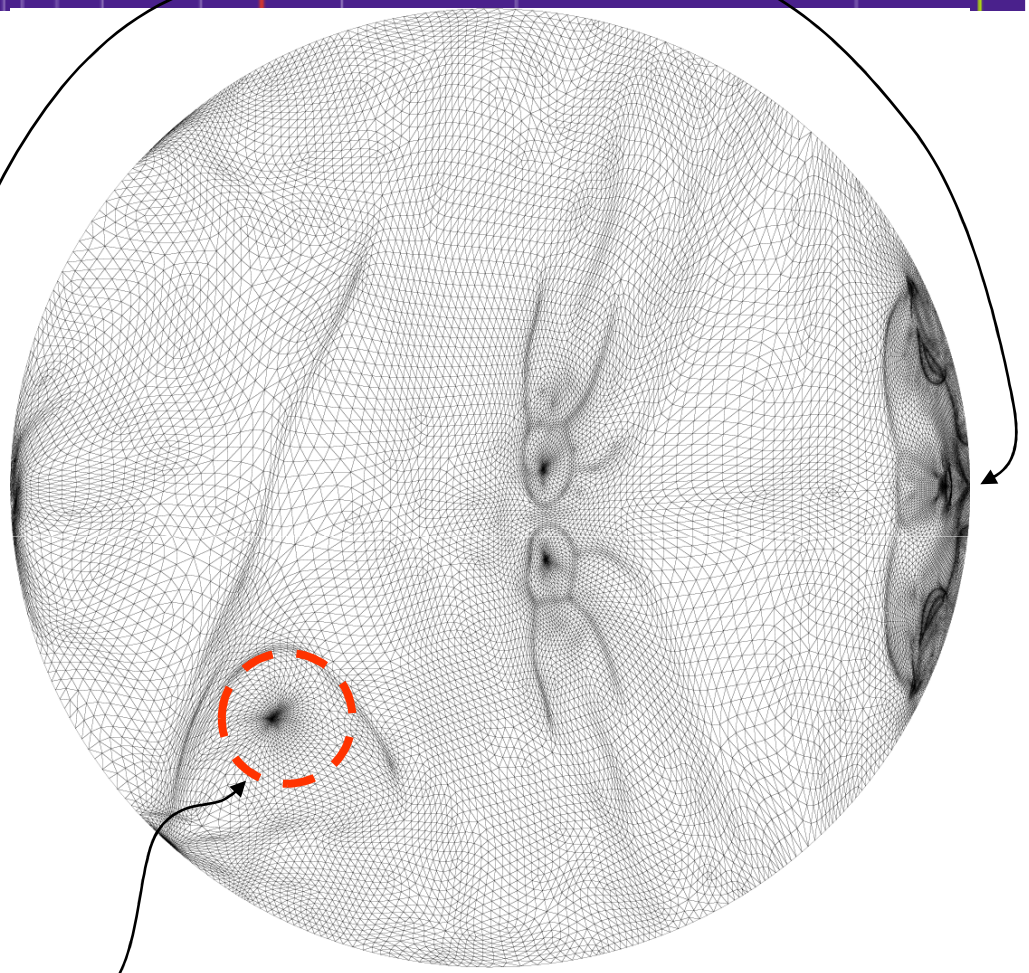


Pierre Alliez INRIA Sophia-Antipolis

A naive cut...

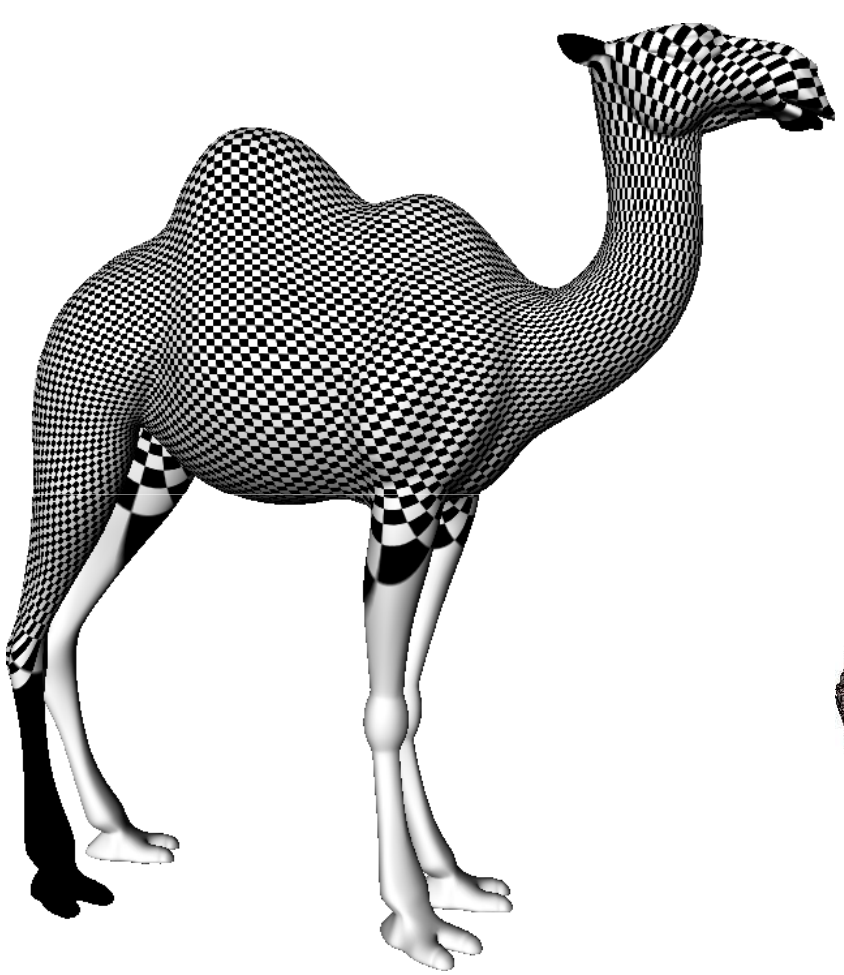


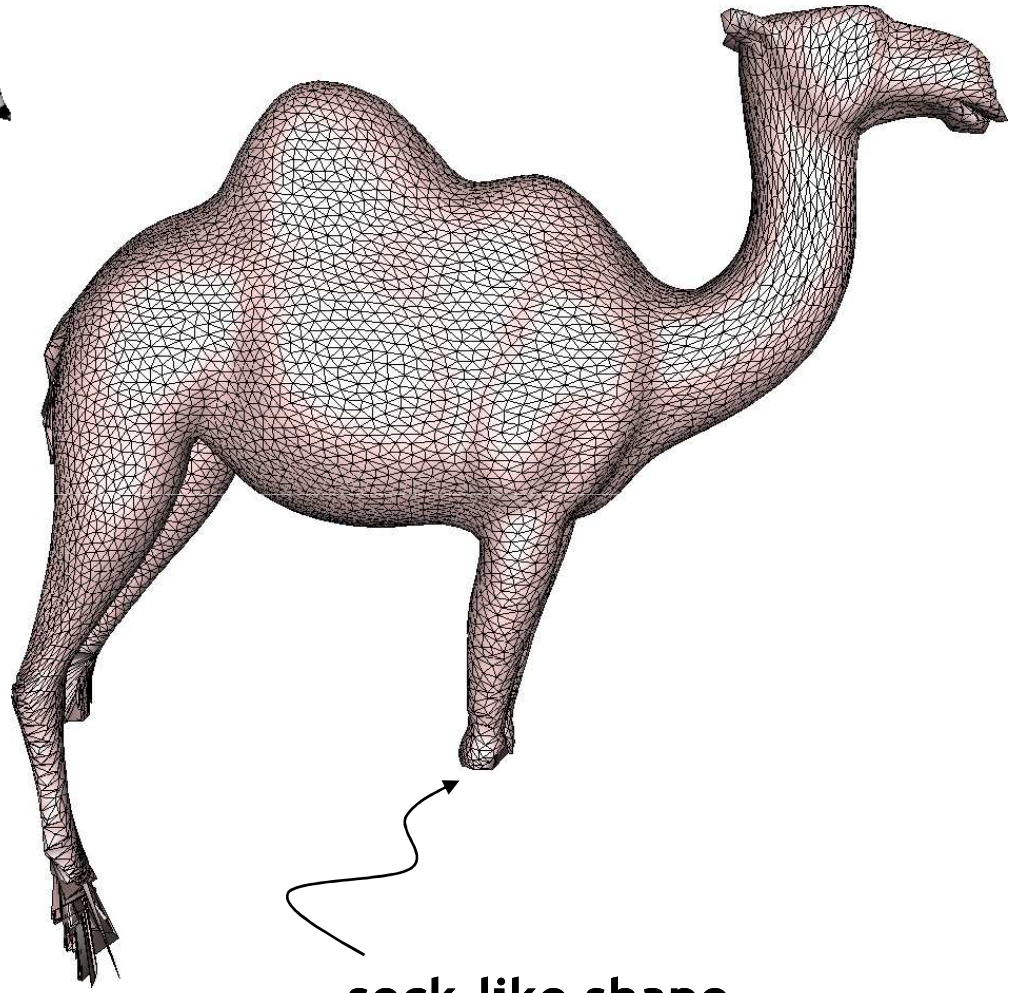




huge isoperimetric distortion

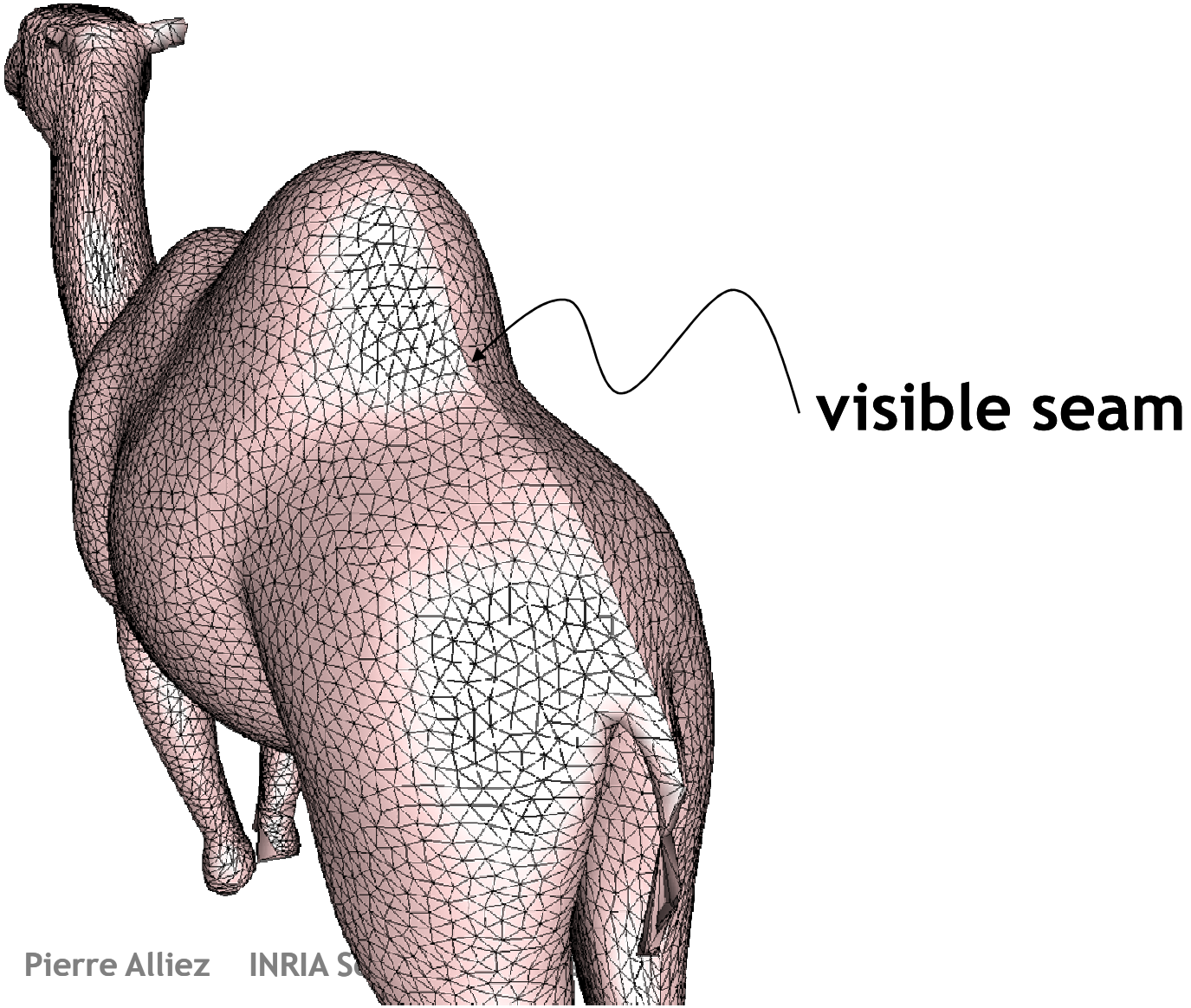
Numerical Issues



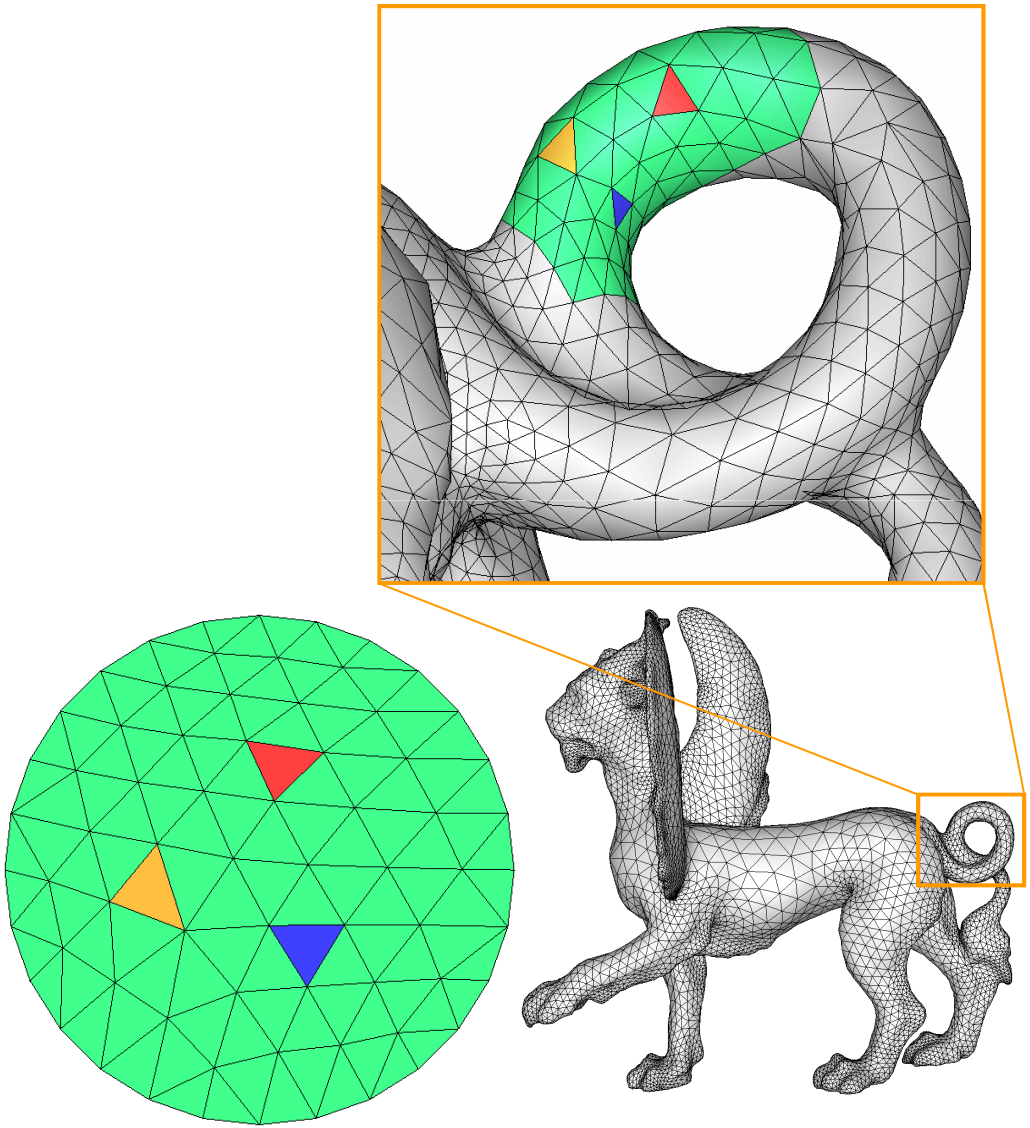


sock-like shape

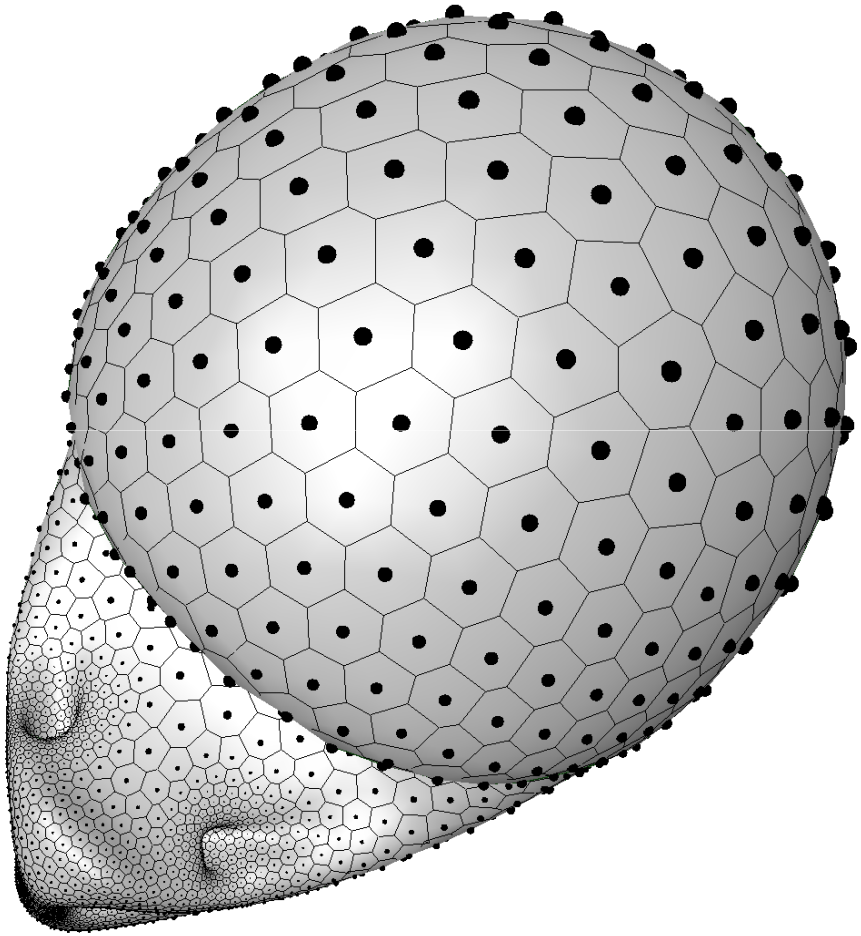
Seaming lines

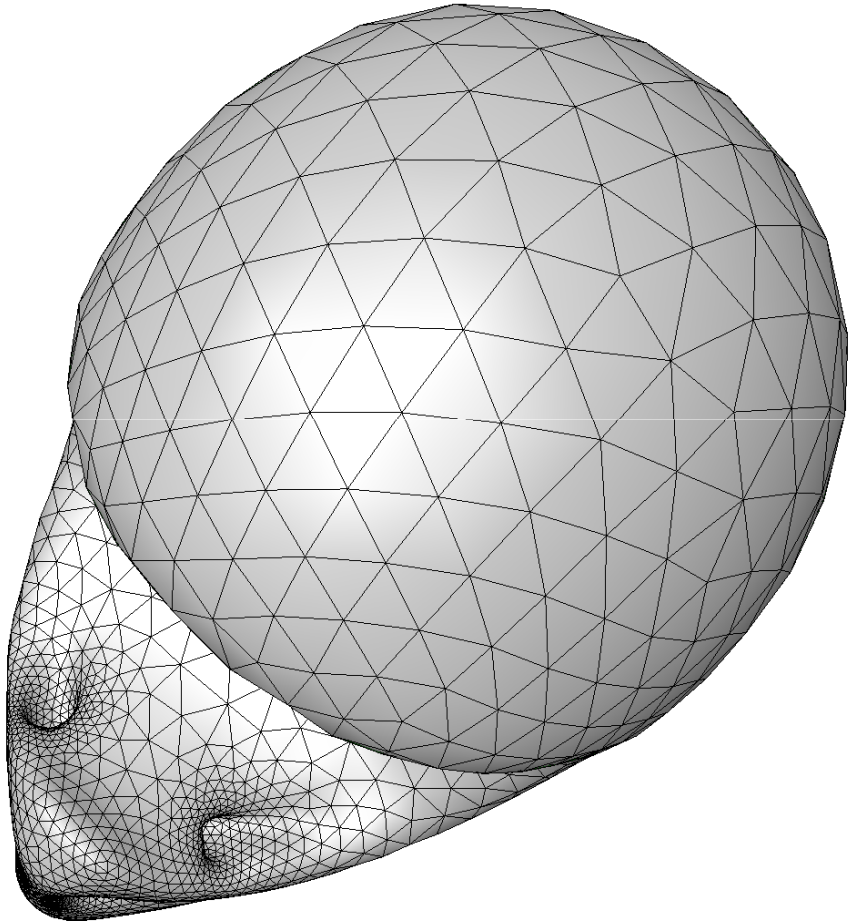


Overlapping Local Parameterizations

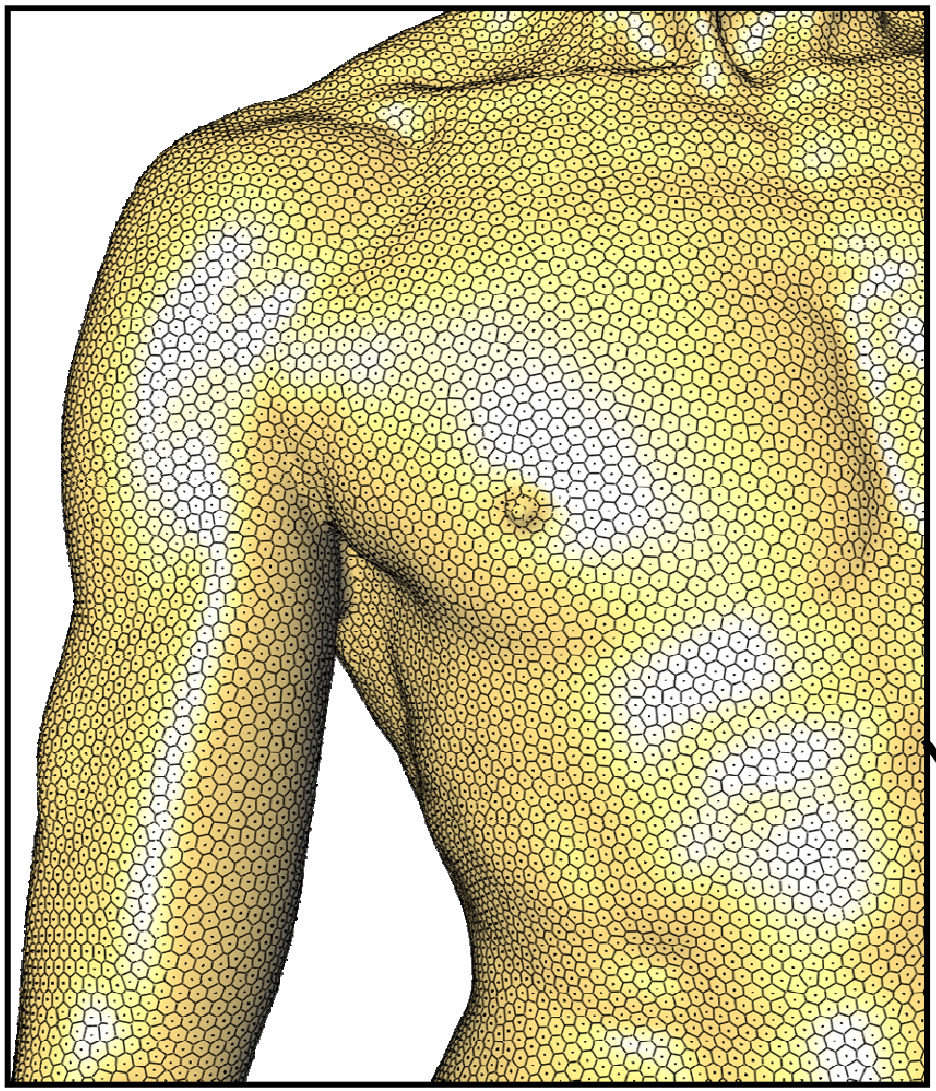


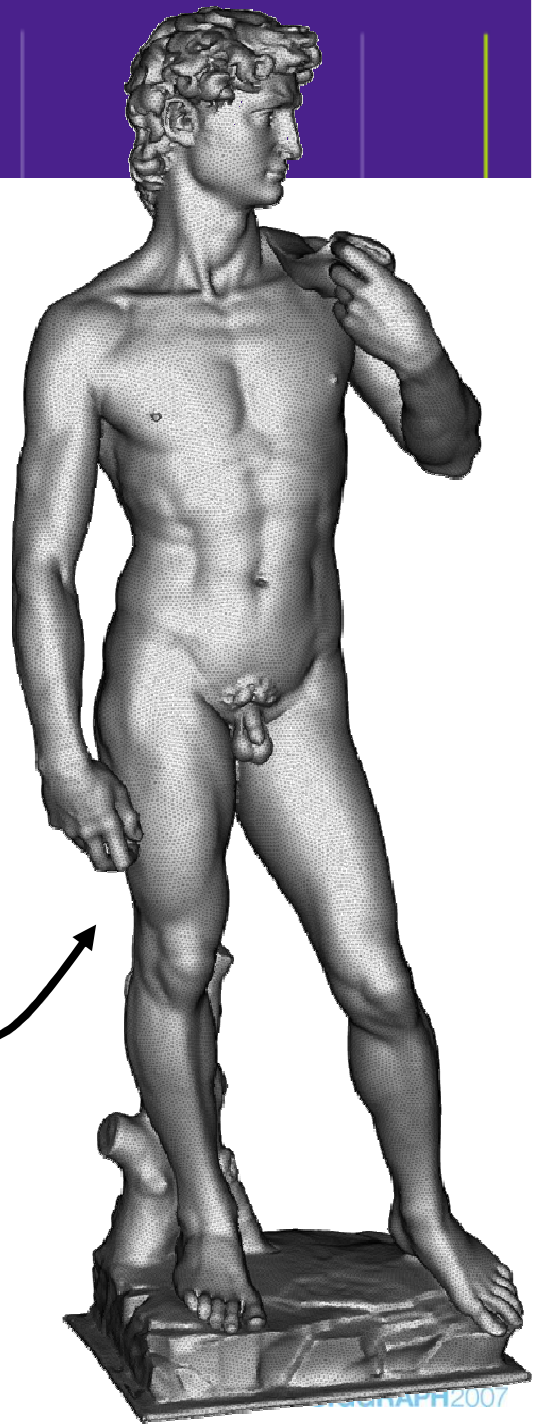
Helmet





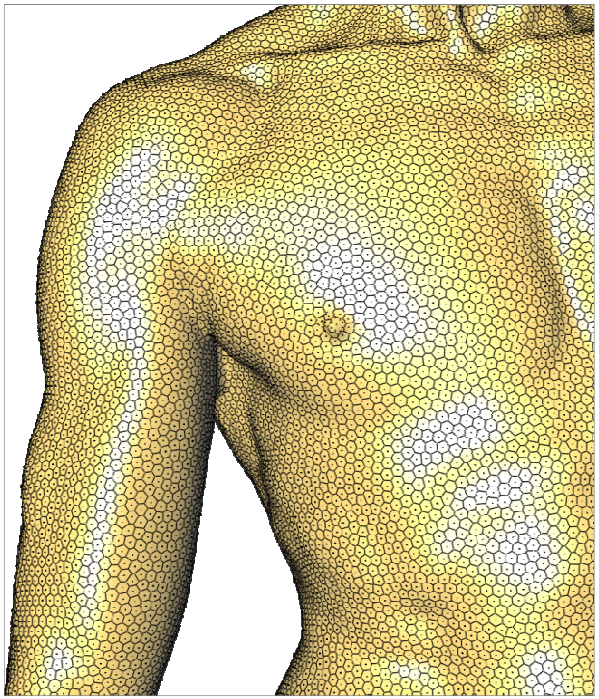
Michelangelo David

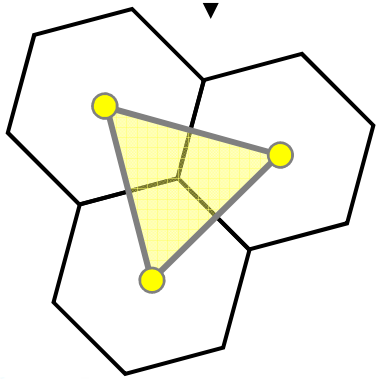


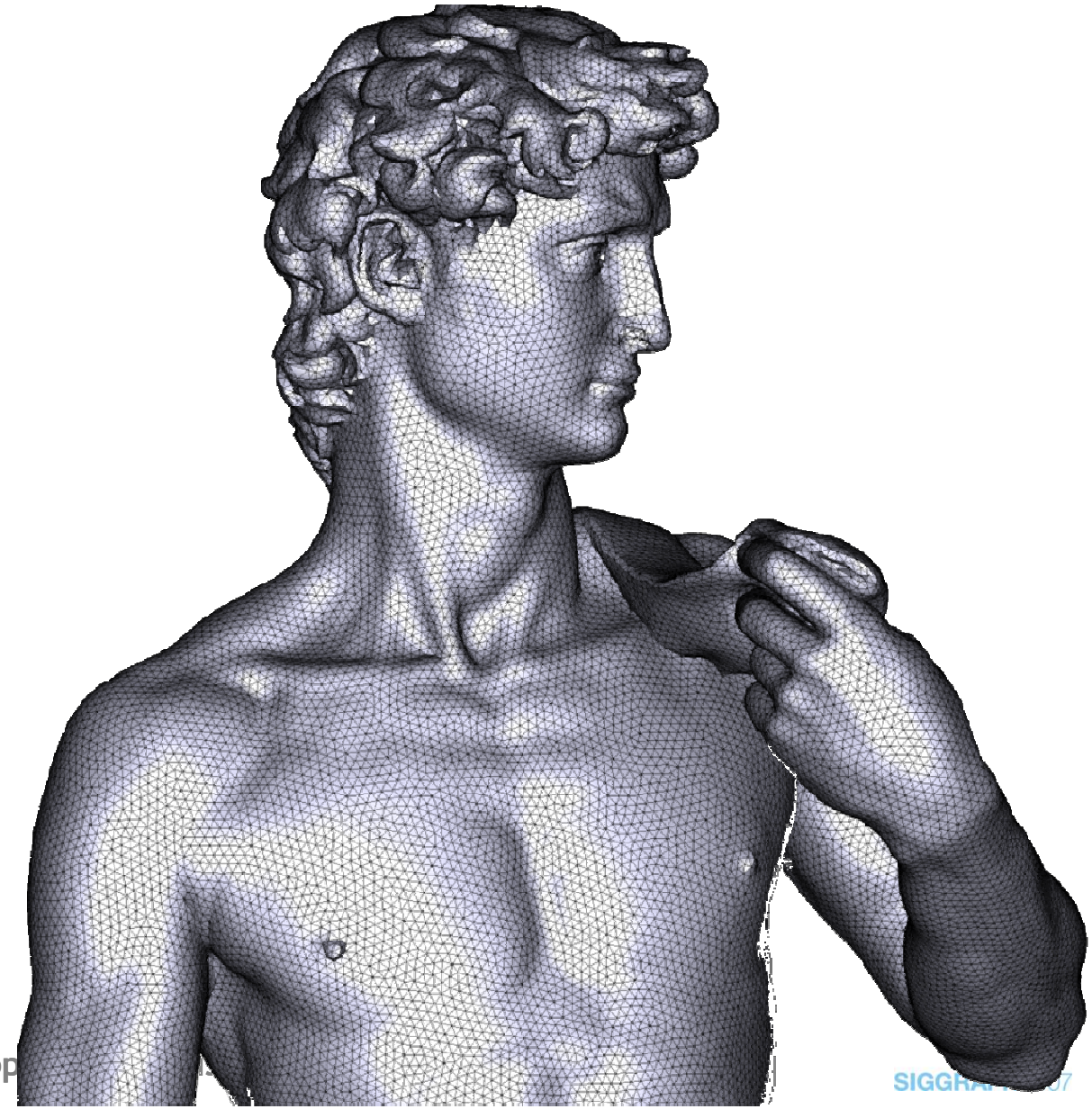


[Surazhsky, A. & Gotsman 03]

Michelangelo David







Pliant Algorithm

Iterative Mesh Optimization

- (area weighted) random scatter
  or simply start with the given mesh
- improve vertex distribution
- update mesh connectivity

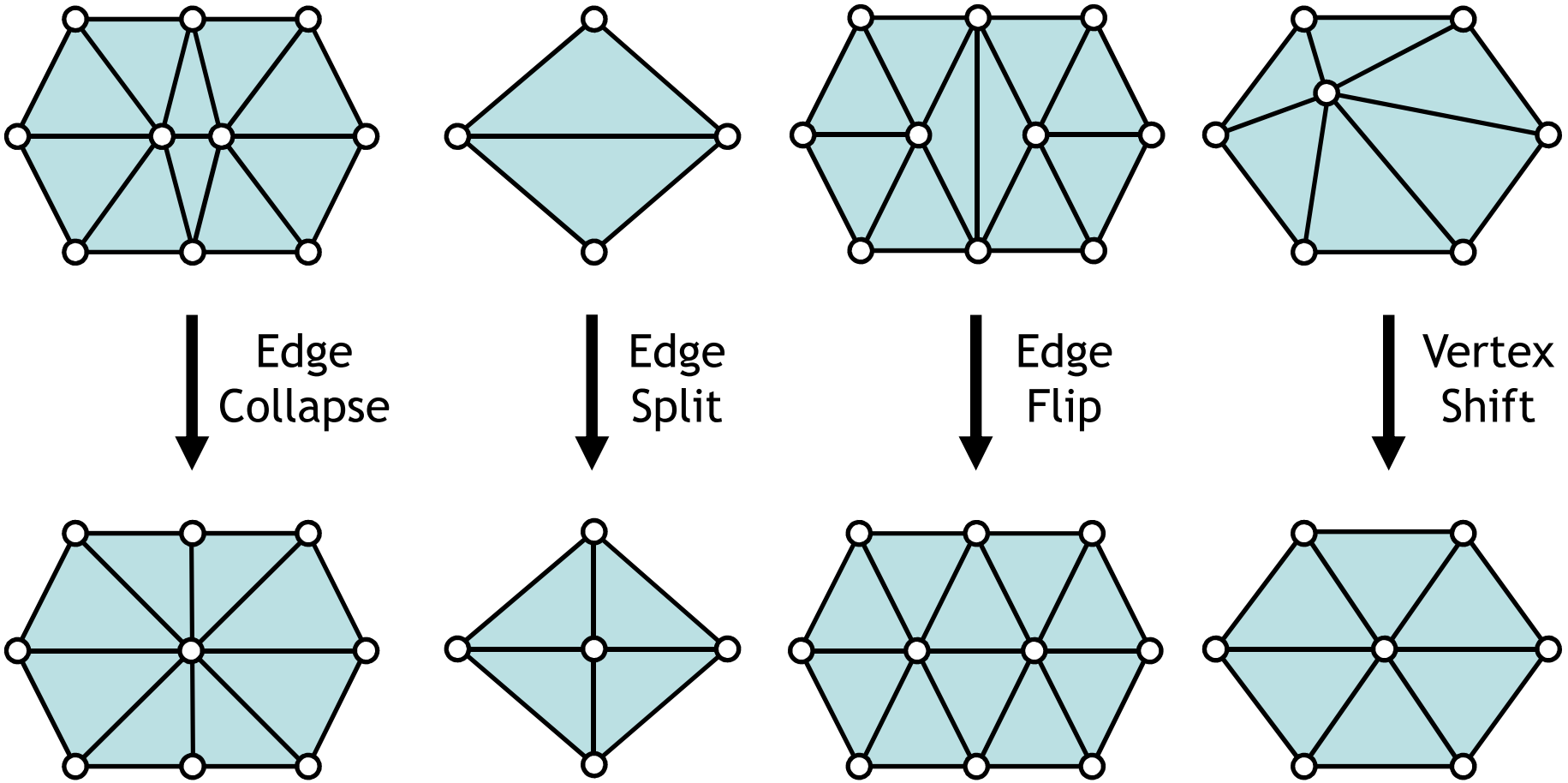
[Botsch-Kobbelt] **SGP 2004, *A remeshing approach to multiresolution modeling*.**

Iterative Mesh Optimization

Isotropic remeshing prefers...

- **Locally uniform edge length**
  - remove too short edges edge collapses
  - remove too long edges 2-4 edge split
- **Regular valences**
  - valence balance edge flip
- **Uniform vertex distribution**
  - tangential smoothing Laplace operator

Local Remeshing Operators



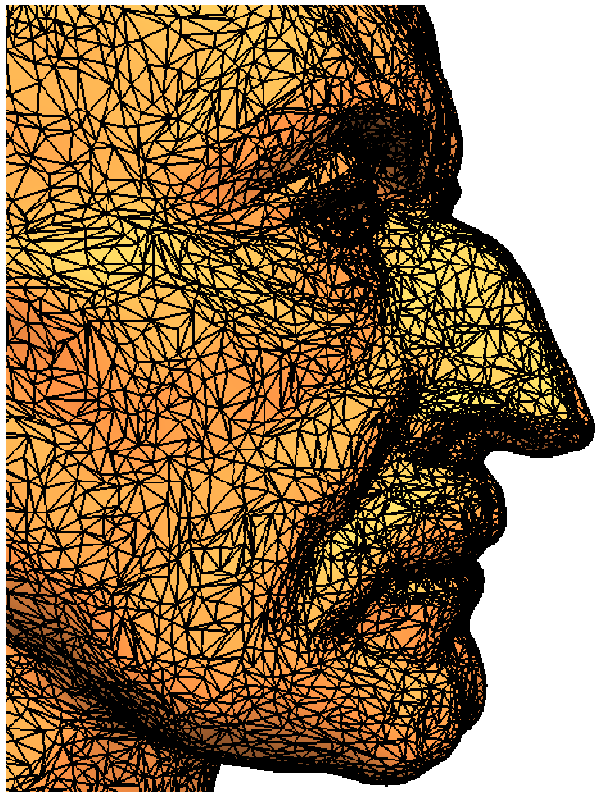
Isotropic Remeshing

Specify target edge length L

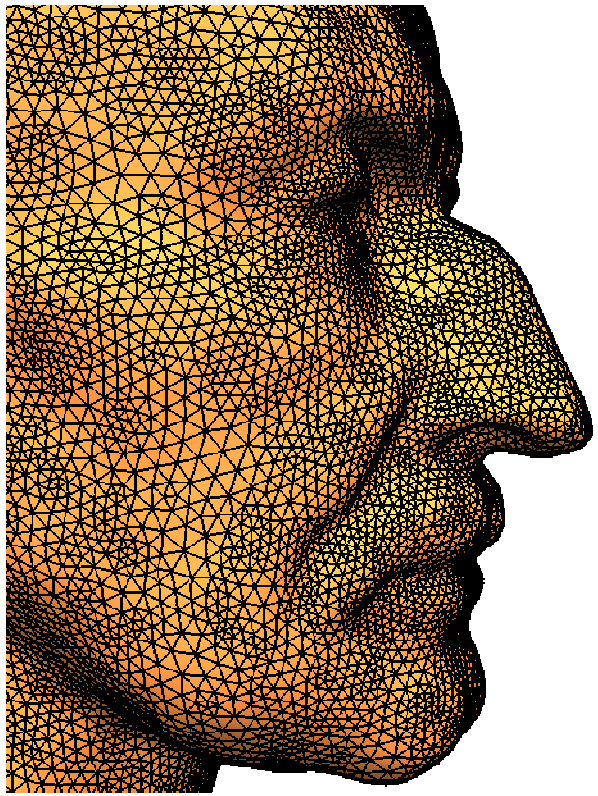
Iterate:

1. **Split** edges longer than L_{max}
2. **Collapse** edges shorter than L_{min}
3. **Flip** edges to get closer to valence 6
4. Vertex **shift** by tangential relaxation
5. **Project** vertices onto reference mesh

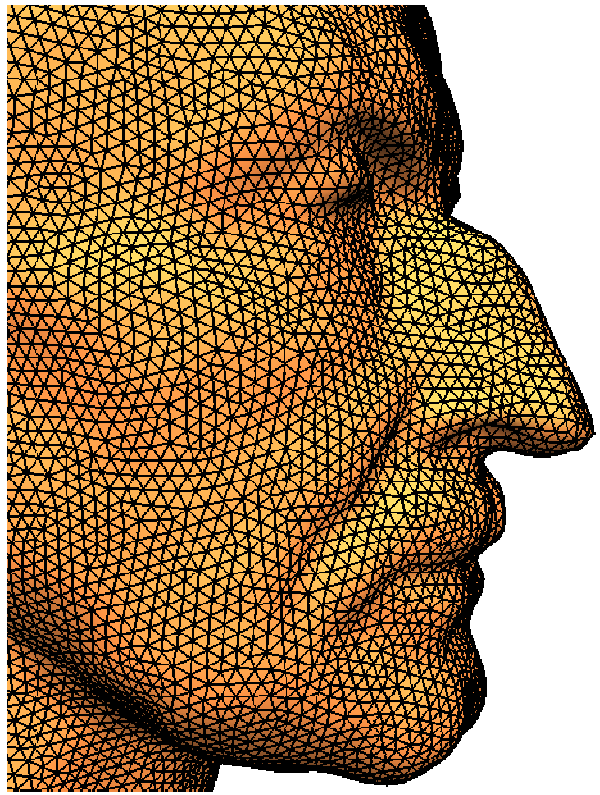
Thresholds L_{min} and L_{max}



Original

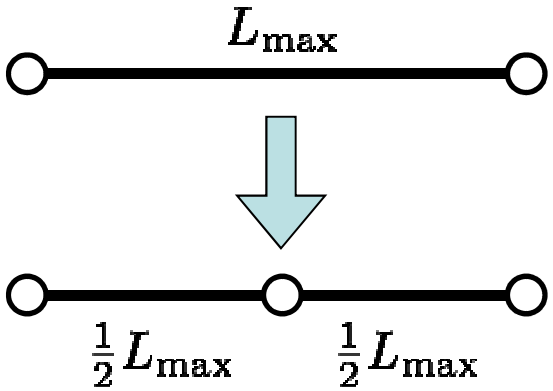


$\left(\frac{1}{2}, 2\right)$

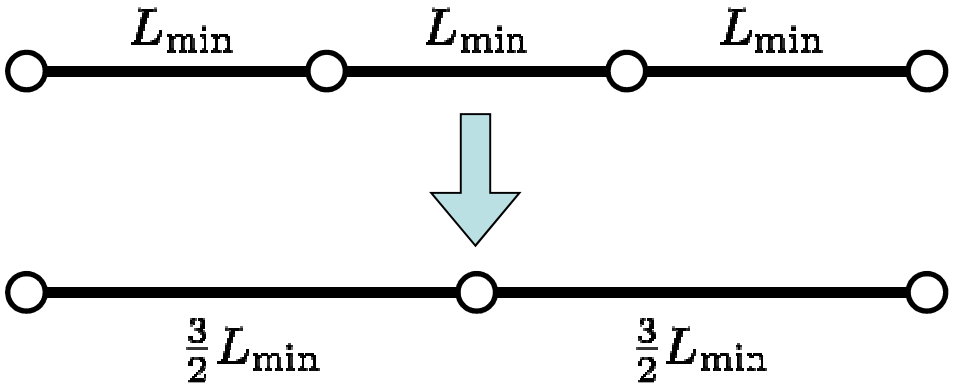


$\left(\frac{4}{5}, \frac{4}{3}\right)$

Edge Collapse / Split



$$|L_{\max} - L| = \left|\tfrac{1}{2}L_{\max} - L\right|$$
$$\Rightarrow L_{\max} = \tfrac{4}{3}L$$

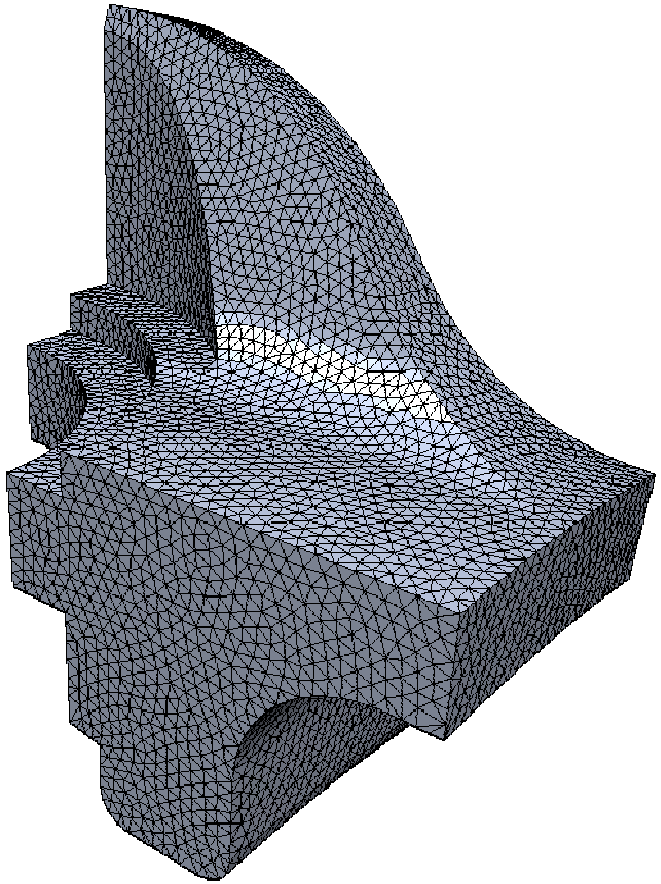


$$|L_{\min} - L| = \left|\tfrac{3}{2}L_{\max} - L\right|$$
$$\Rightarrow L_{\min} = \tfrac{4}{5}L$$

demo

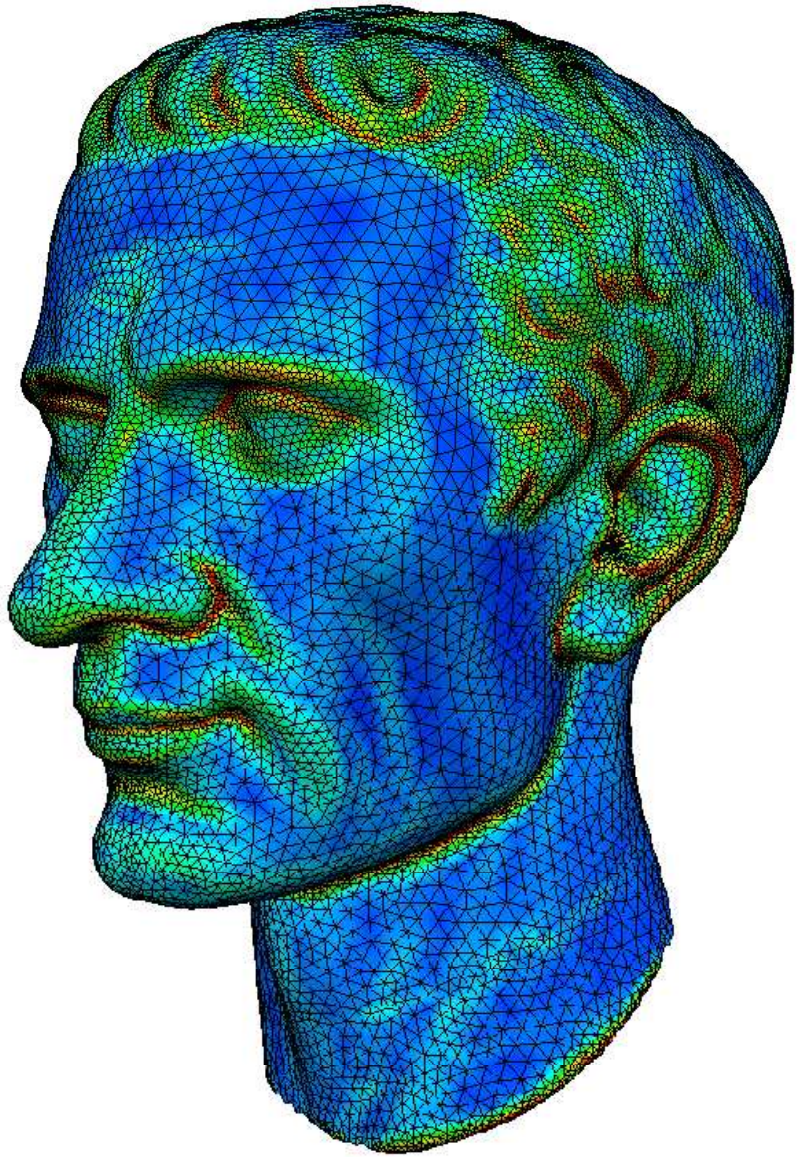
Feature Preservation

- **Define features**
  - sharp edges
  - material boundaries
- **Adjust local operators**
  - do not flip
  - collapse only along features
  - univariate smoothing
  - project to feature curves



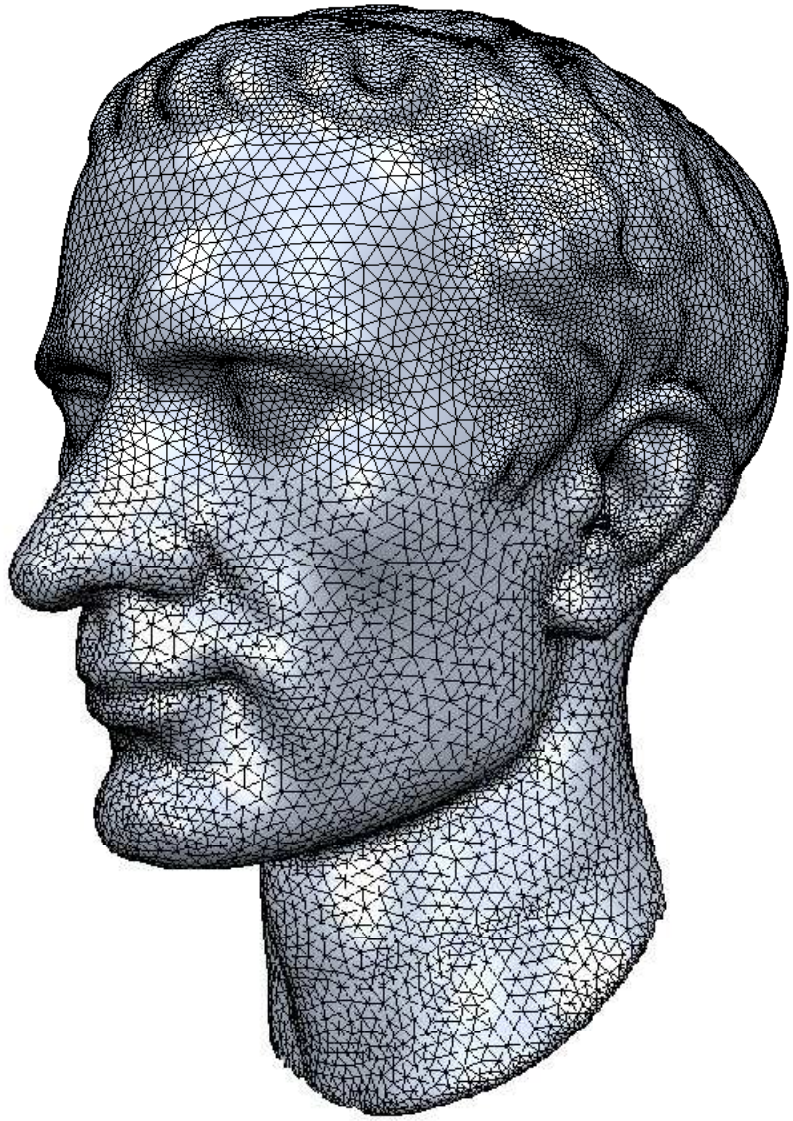
Adaptive Remeshing

- target edge length locally determined by curvature
- adjust split / collapse criteria



Isotropic Remeshing

- **High quality triangulations**
  - Well-shaped triangles
  - Valence 6
- **Extensions**
  - feature preservation
  - curvature adaptation
- **Local operators & projection**
  - easy to implement
  - computationally efficient
  - 100K vertices in < 5 sec



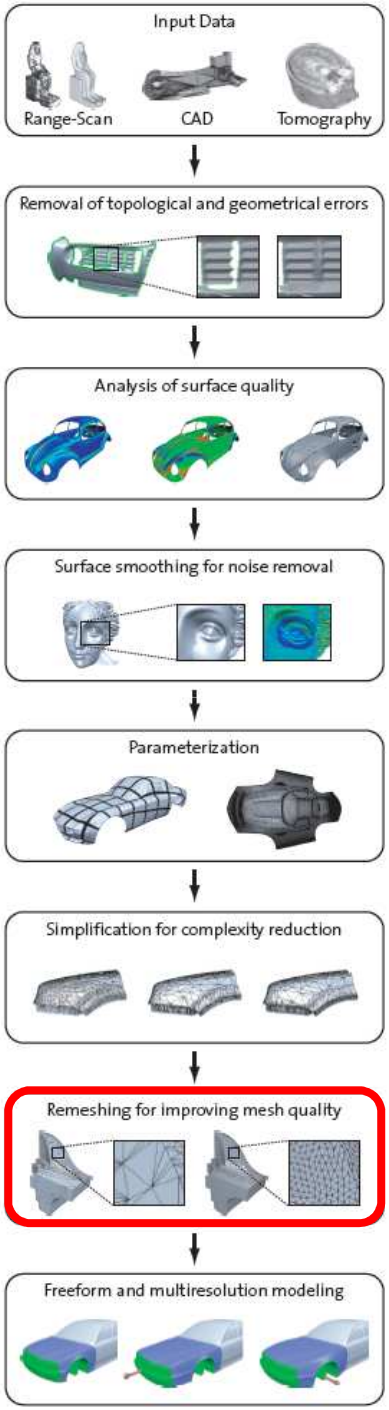
Summary

Paradigms:

- Greedy
- Variational
- Pliant

Three approaches:

- **Ambiant space**
  - restricted Delaunay triangulation
  - interpolant
  - no self-intersection
- **Surface oriented**
  - operate directly of the surface
  - efficient for high resolution remeshing (locally flat surface)
- **Parameterization-based**
  - map to 2D domain / 2D problem
  - often less efficient
  - works even for coarse resolution remeshing

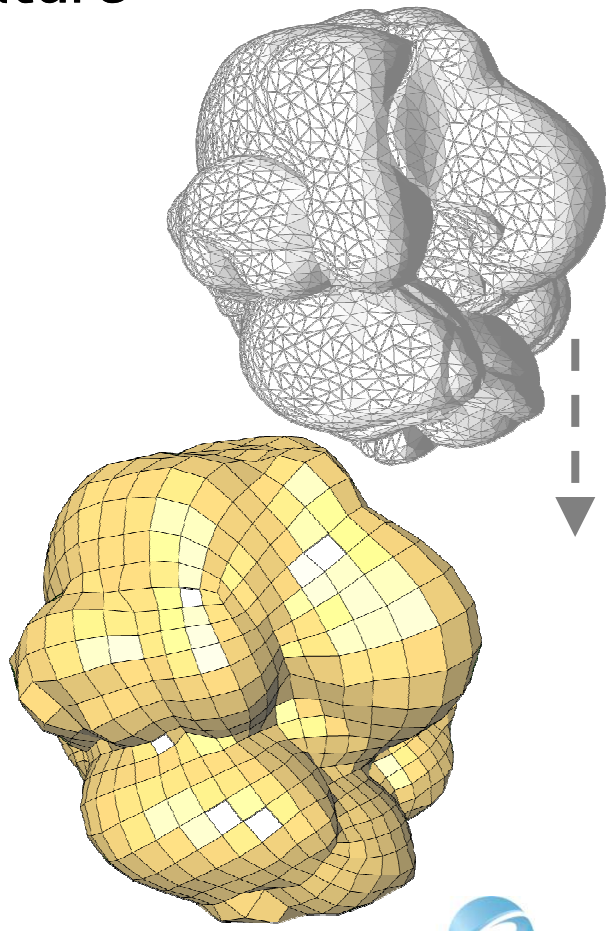


Quadrangle Remeshing

Quadrangulations

- Needed in CG, CAGD, Reverse Engineering
  - Ubiquitous due to tensor-product nature
    - Modeling anisotropy/symmetries
    - B-spline fitting, FEM, texture atlasing
  - But global topology constraints…
- **A Variety of Requirements:**
  - Isotropy *vs* anisotropy
  - Orthogonality

  } shape of elements
  - Alignment (approx. or semantic)
  - Regularity
  - Sizing

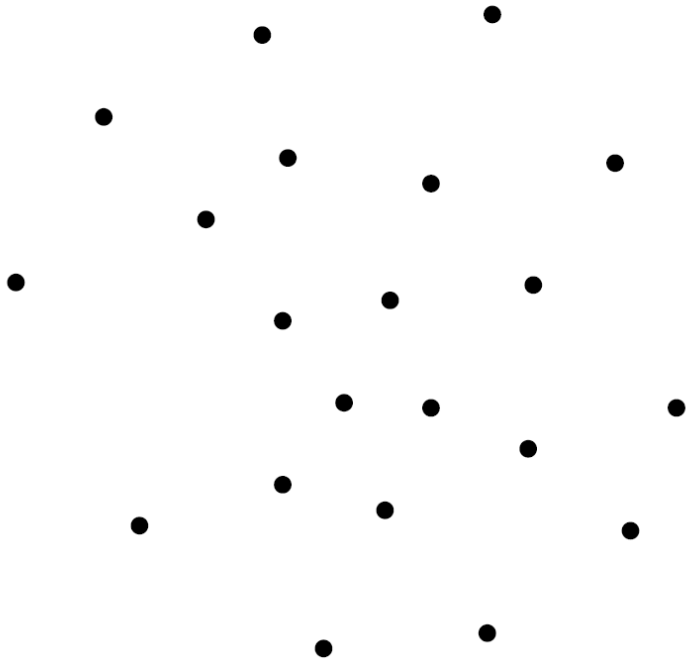


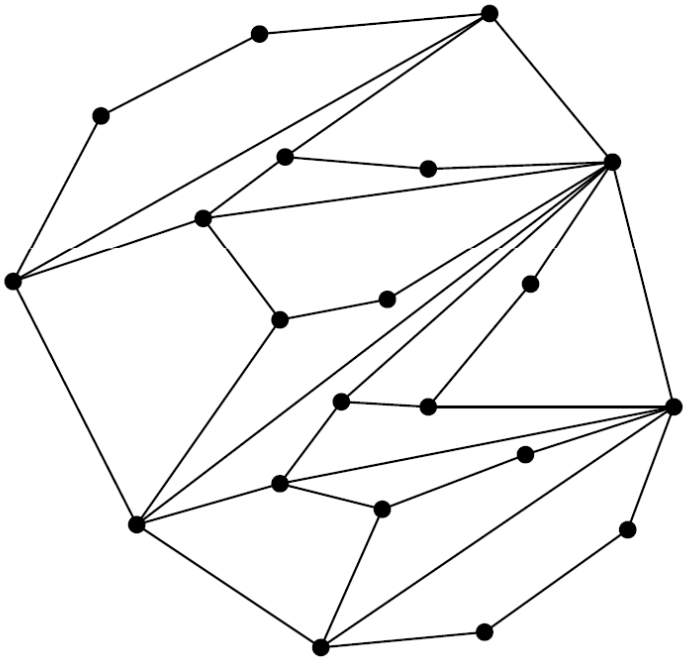
Main Paradigms

- Quadrangulating Point Sets
- Conversion
- Curve-based Sampling
- Contouring

Quadrangulating Point Sets

Quadrangulating 2D Point Sets



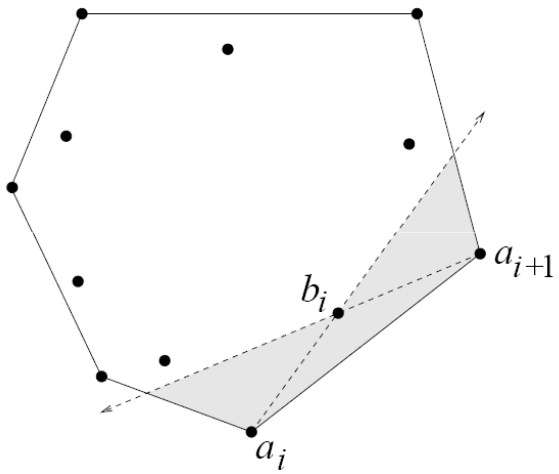


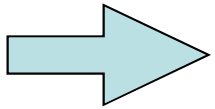
Convex-Quadrangulations of Point Sets

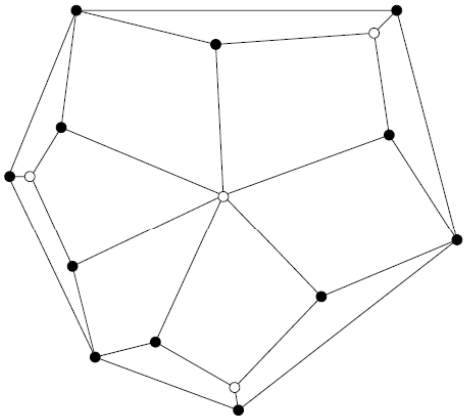
- Each quad must be **strictly convex**

- **Issues:**
  - bounds on #Steiner points
  - constructive algorithms
  - …better than convex
  - [Toussaint, Rappaport, Hurtado, Bremner, Ramaswami…]

Convex-Quadrangulations of Point Sets





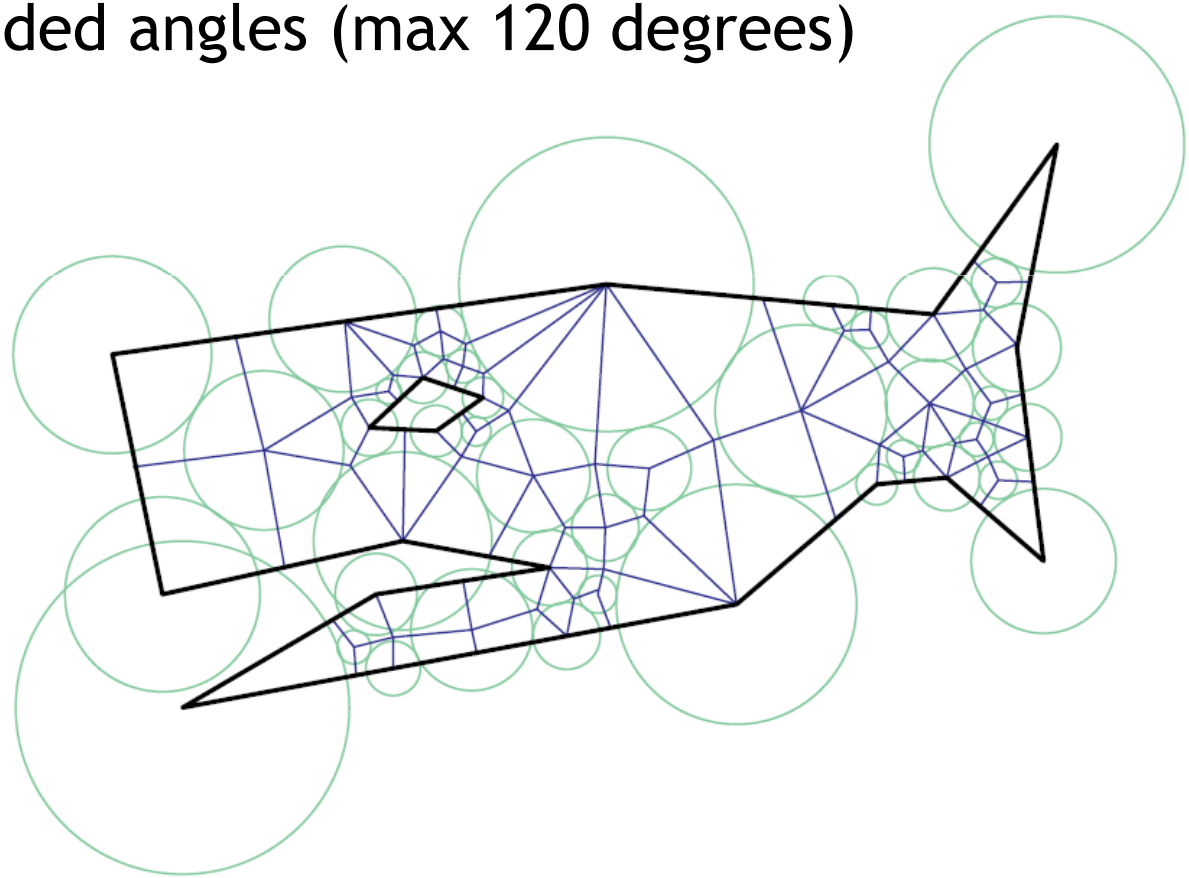


A convex-quadrangulation of P that uses $\frac{n}{4} + 1$ Steiner points.

[Bremner et al. 02]

Quadrangle Meshing

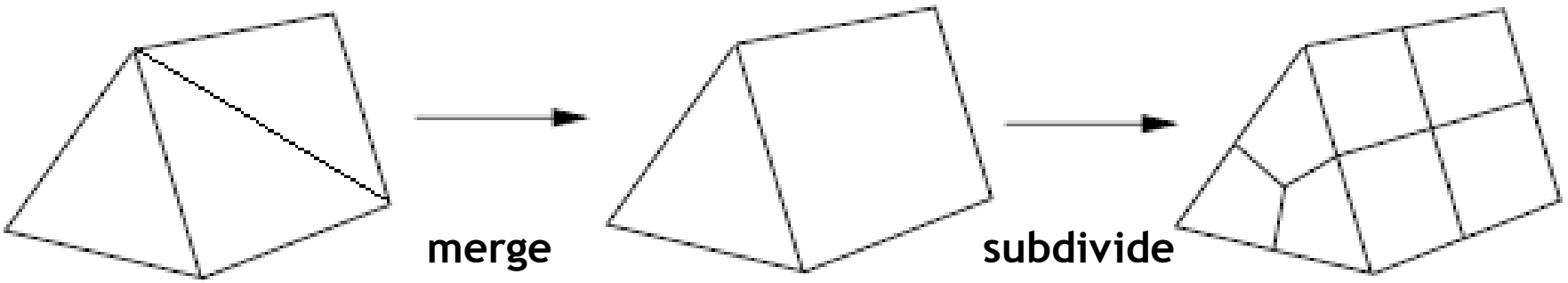
- ***Quadrilateral Meshing by Circle Packing.*** **[Bern & Eppstein 97] IMRT**
  - Bounded angles (max 120 degrees)

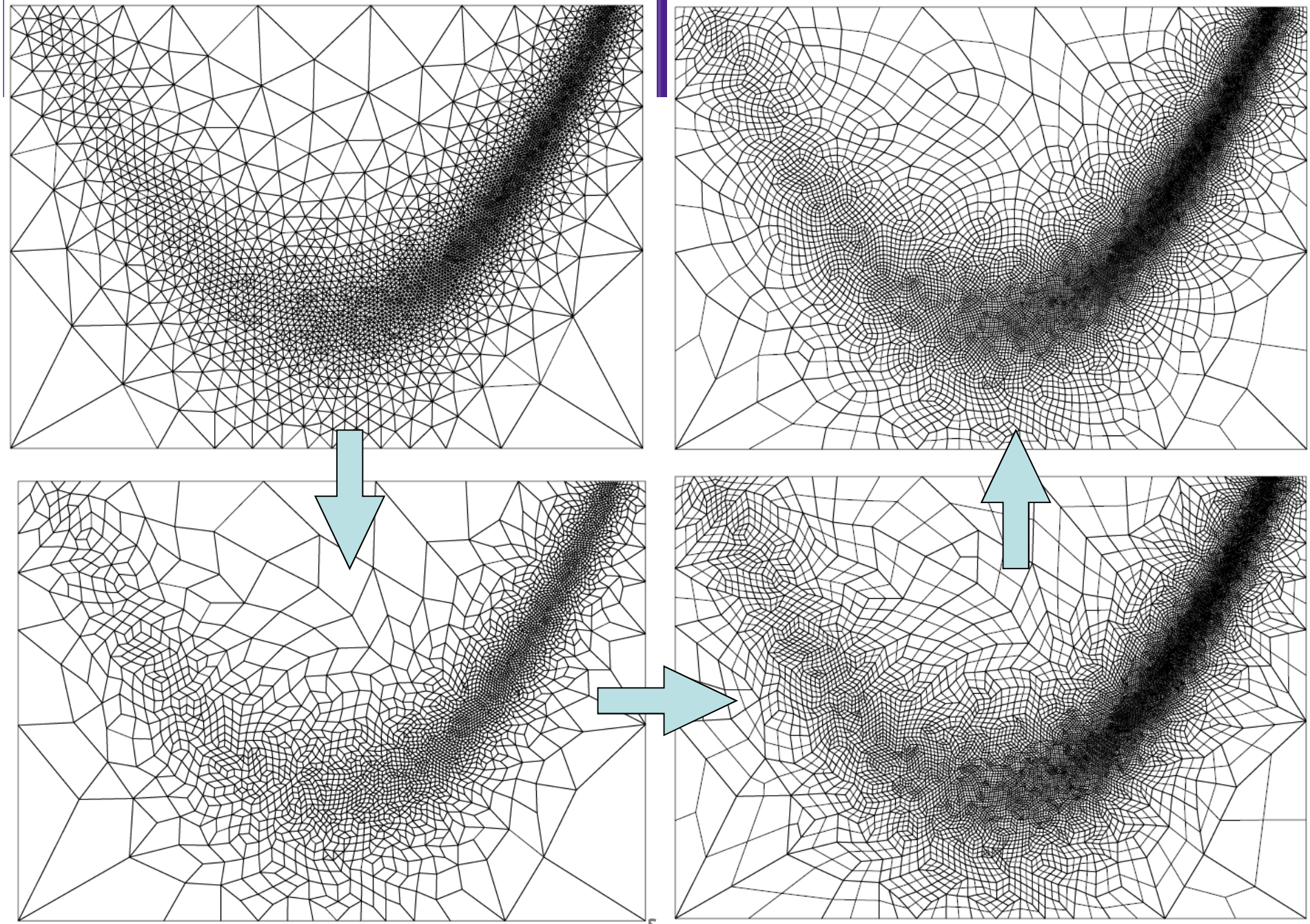


Conversion

Quadrangle Meshing

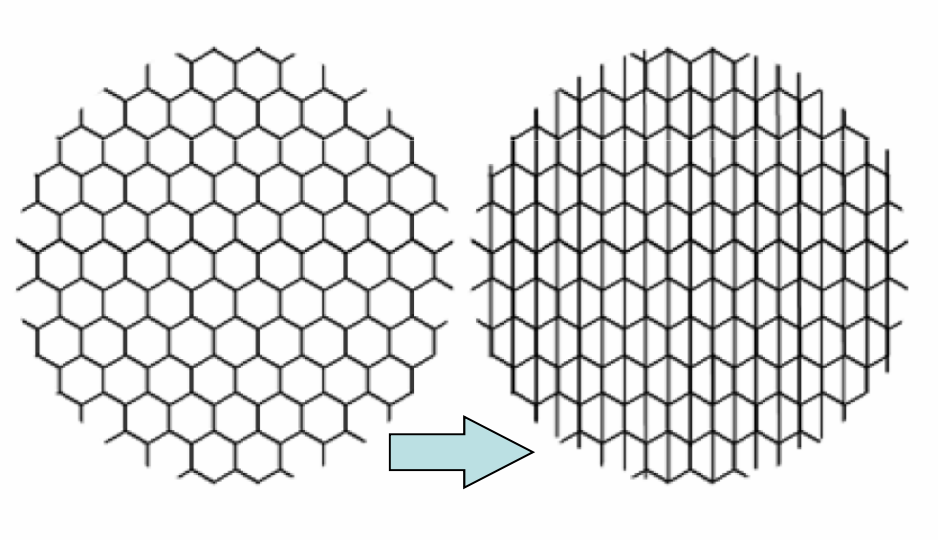
- [Borouchaki-Frey 96] *Adaptive Triangular-Quadrilateral Mesh Generation.*



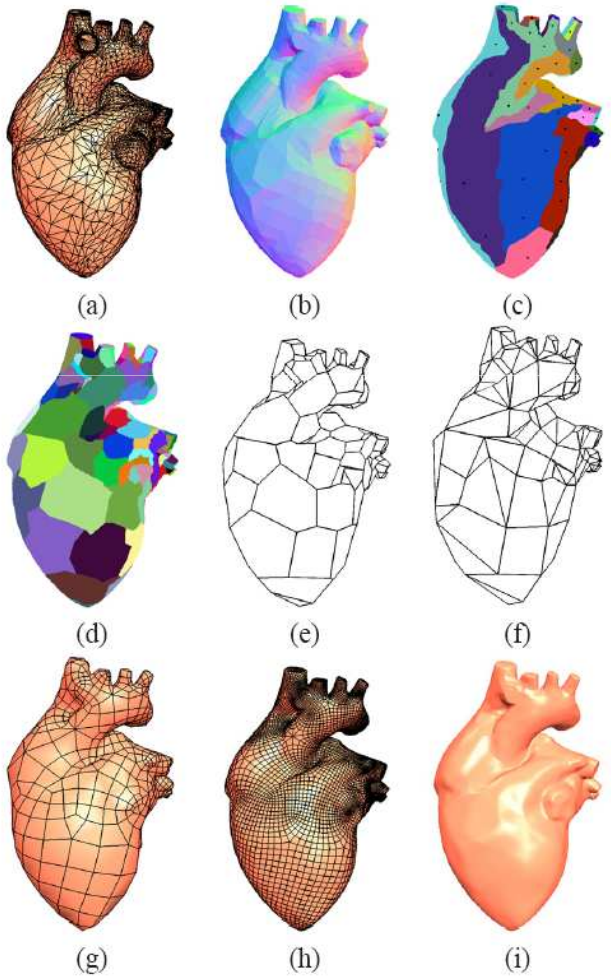


Clustering

- *Parameterization of Triangle Meshes over Quadrilateral Domains*
  [Boier-Martin et al.] SGP 04



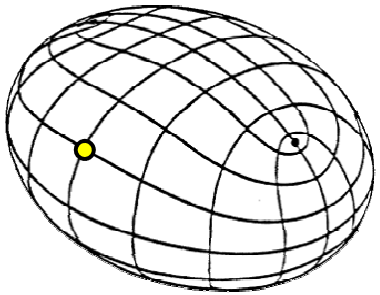
bisection



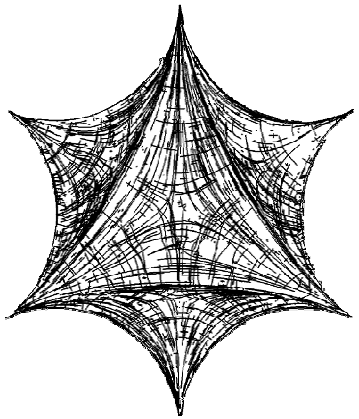
Curve-based Sampling

Curve-based Sampling?

Lines of curvatures for shape depiction

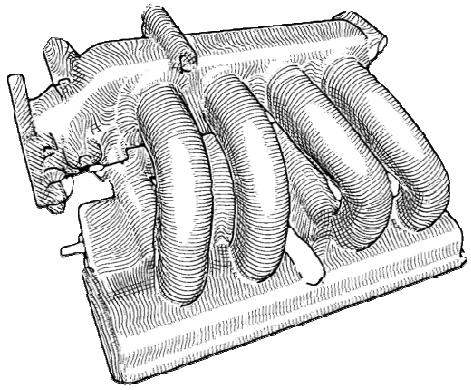


Bradly et al. 85



Elber 98

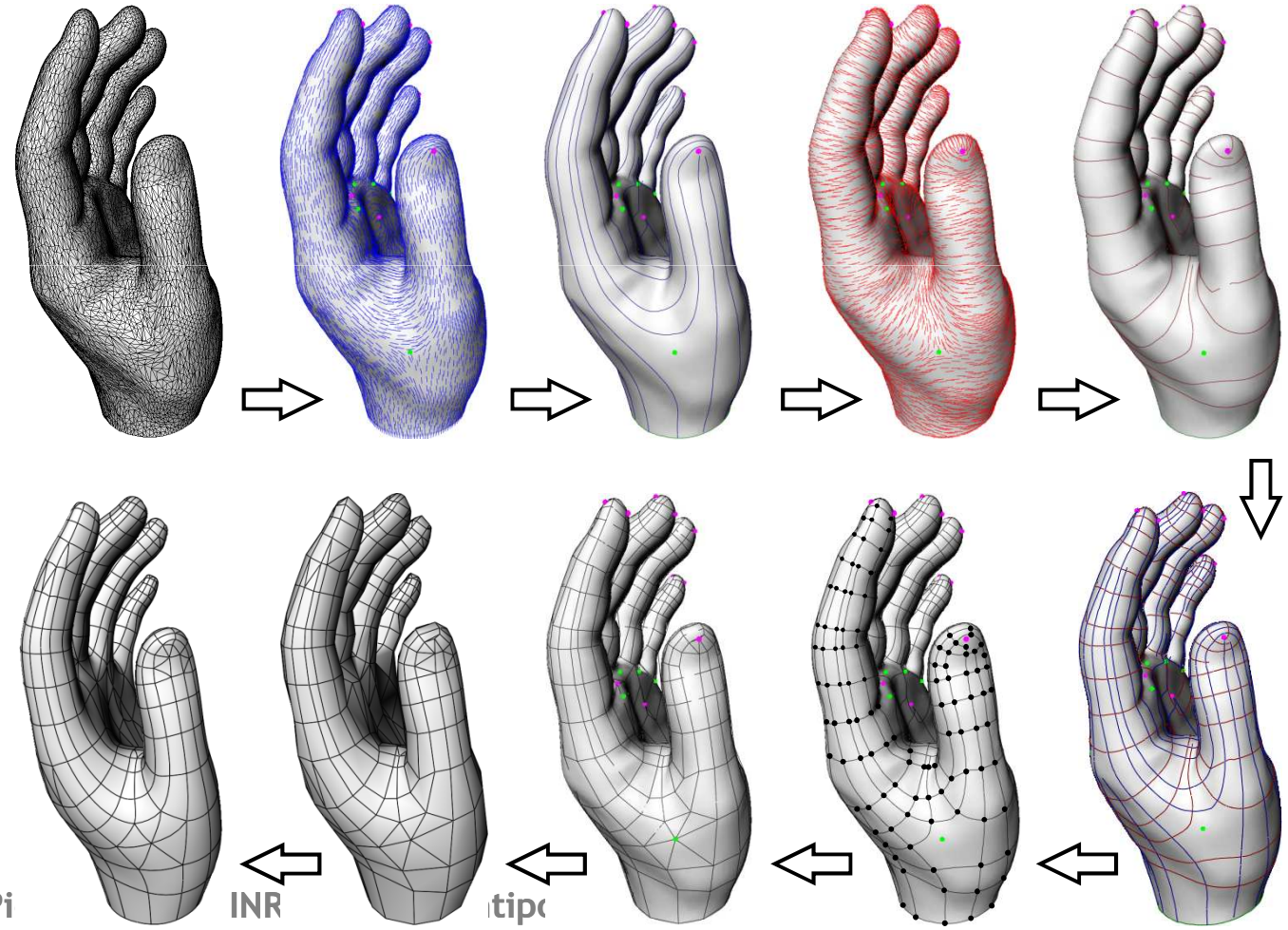
Hertzmann & Zorin 00



Rössl & Kobbelt 01

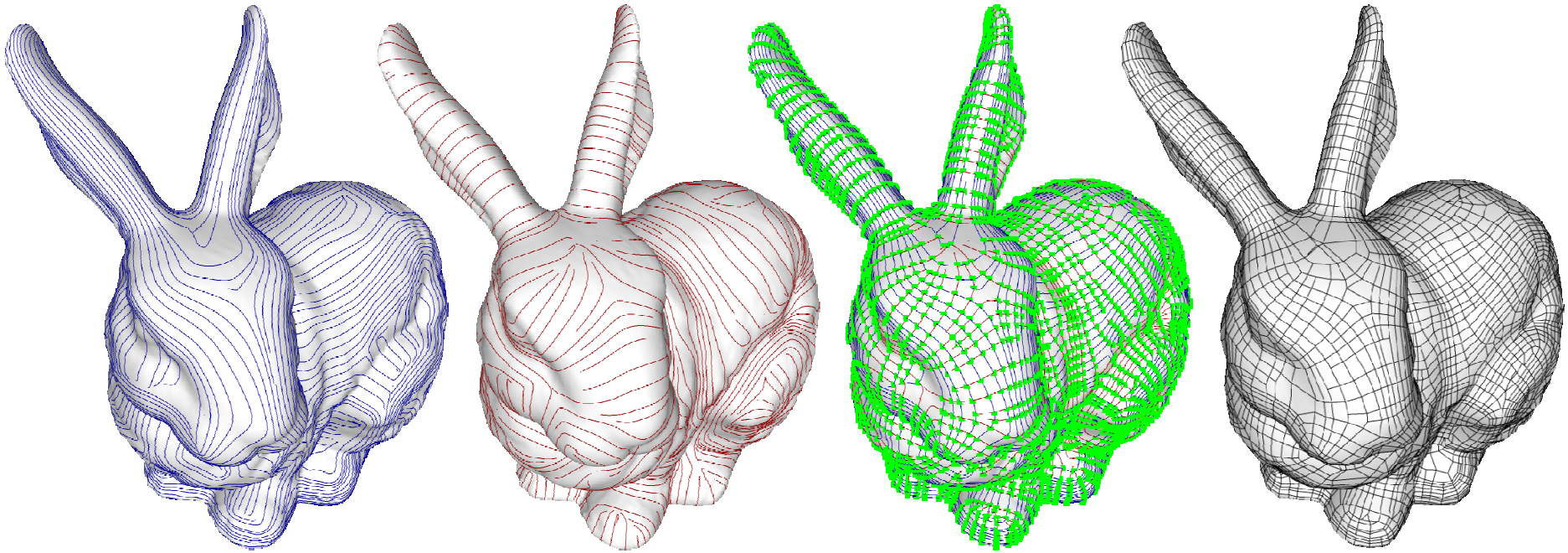
Curve-based Sampling

[A., Cohen-Steiner, Devillers, Desbrun, Levy 2003] SIG 2003, *Anisotropic Polygonal Remeshing*



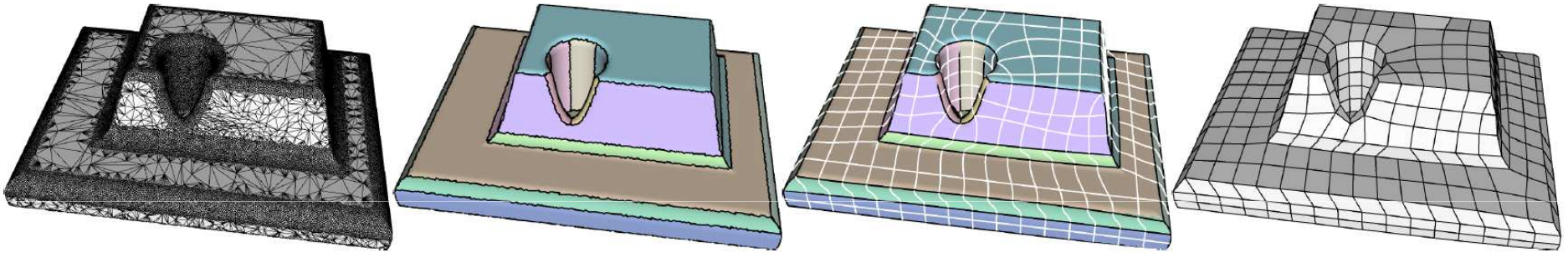
Direction Fields

- *Direct Anisotropic Quad-Dominant Remeshing* [Marinov & Kobbelt 04]
  - No need for parameterization



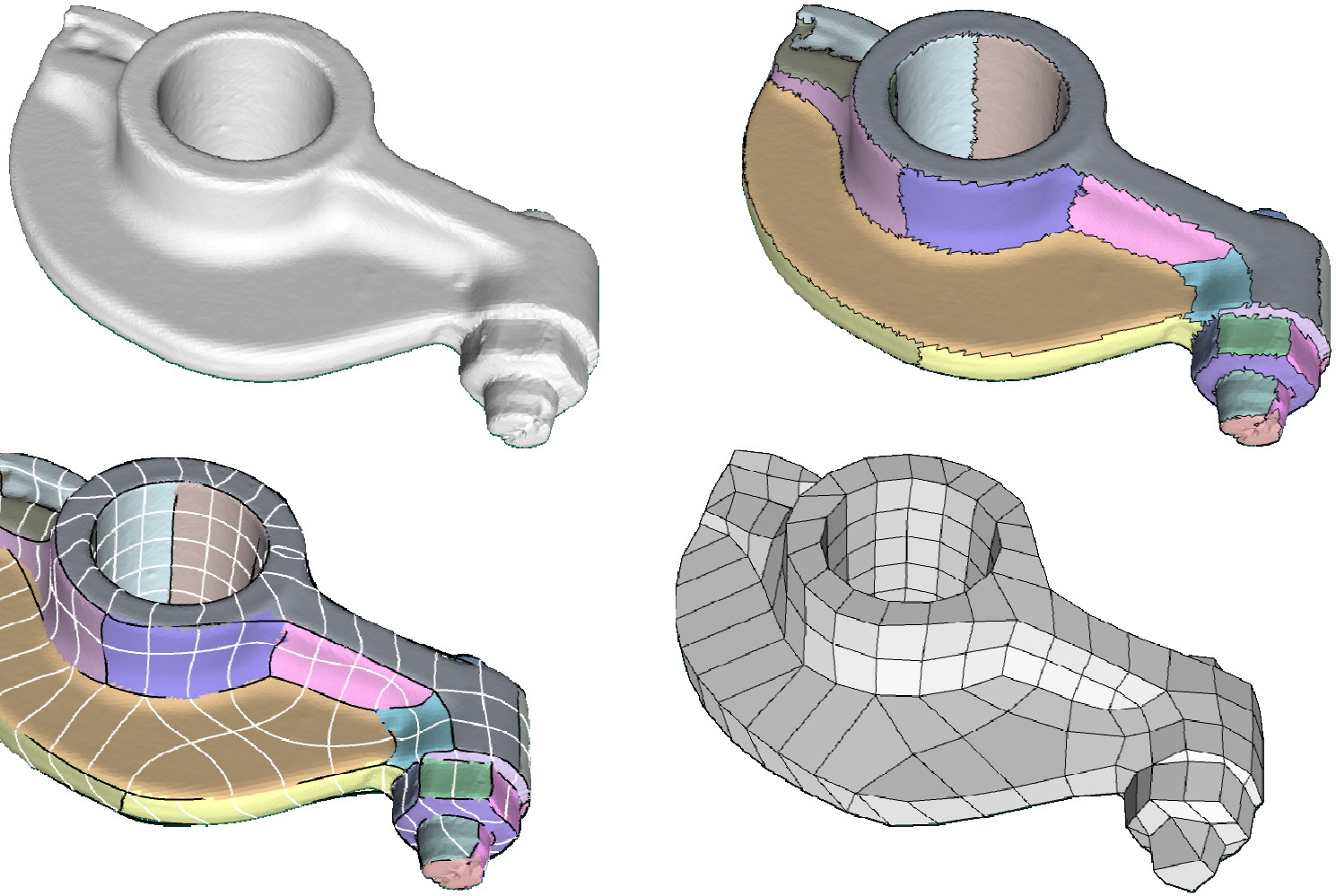
Curve-based Sampling

- Segmentation & Curve Network [Marinov-Kobbelt 06]

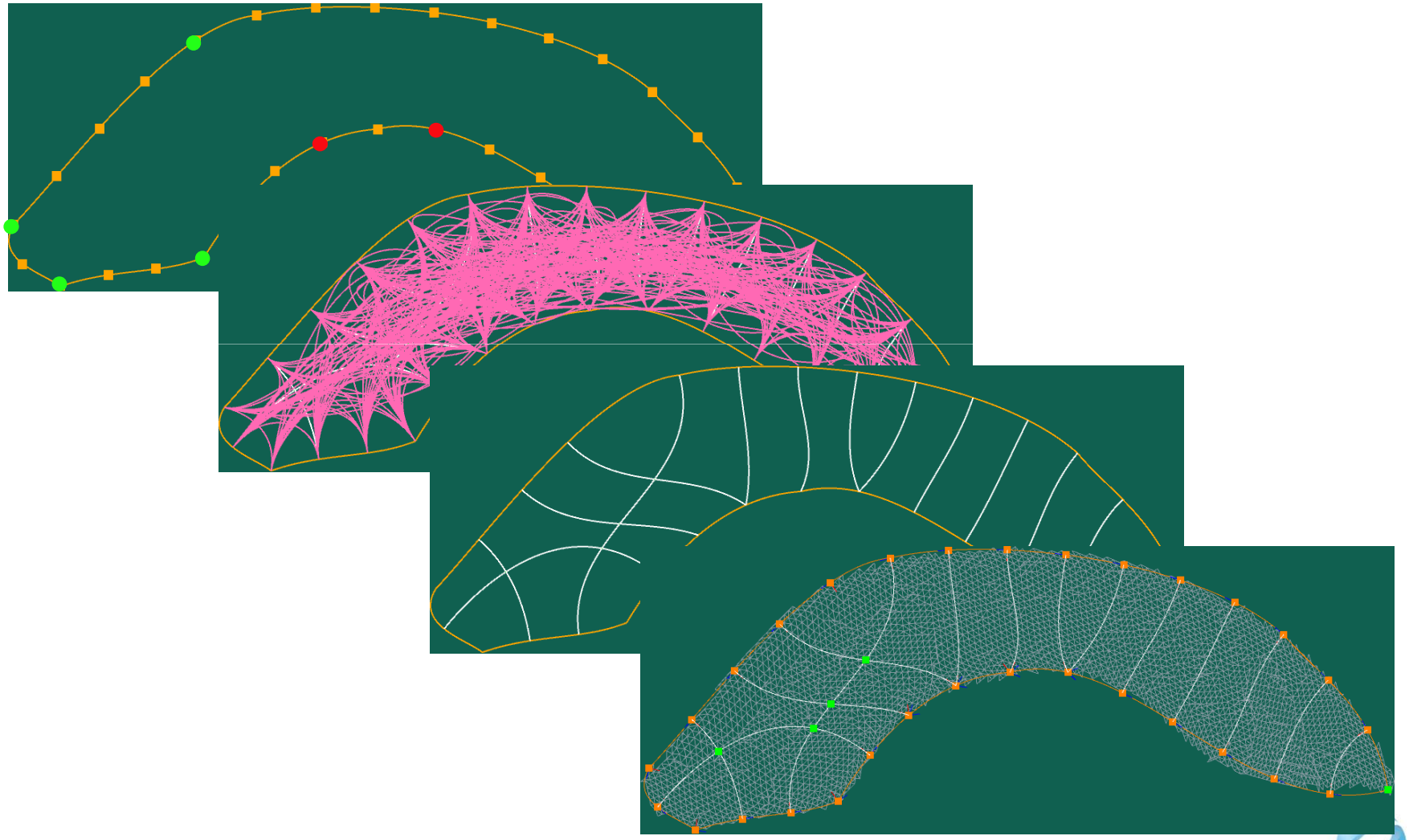


Minimum bending energy curves

Global Alignment

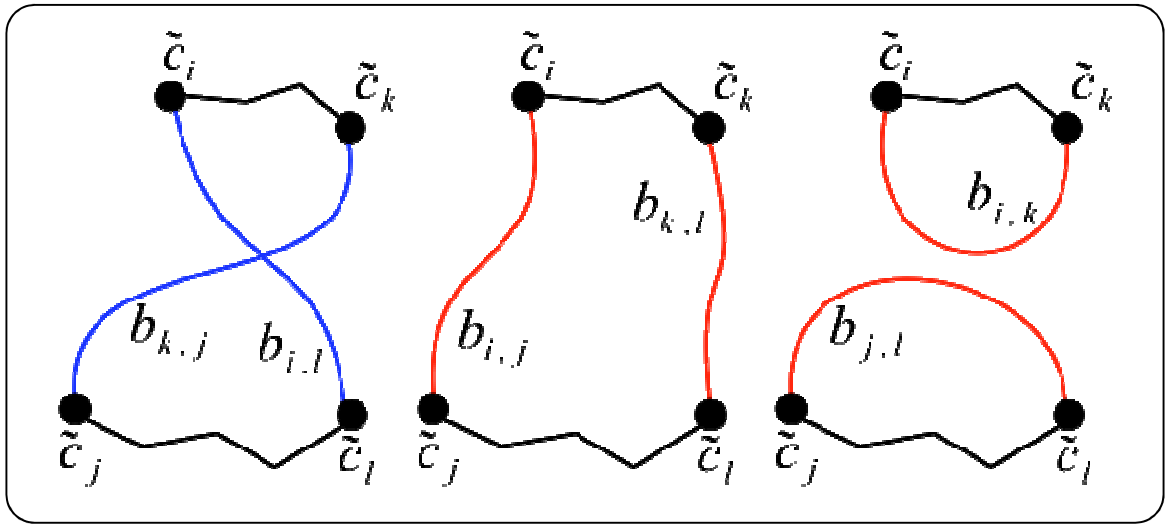


Per-Segment Optimization

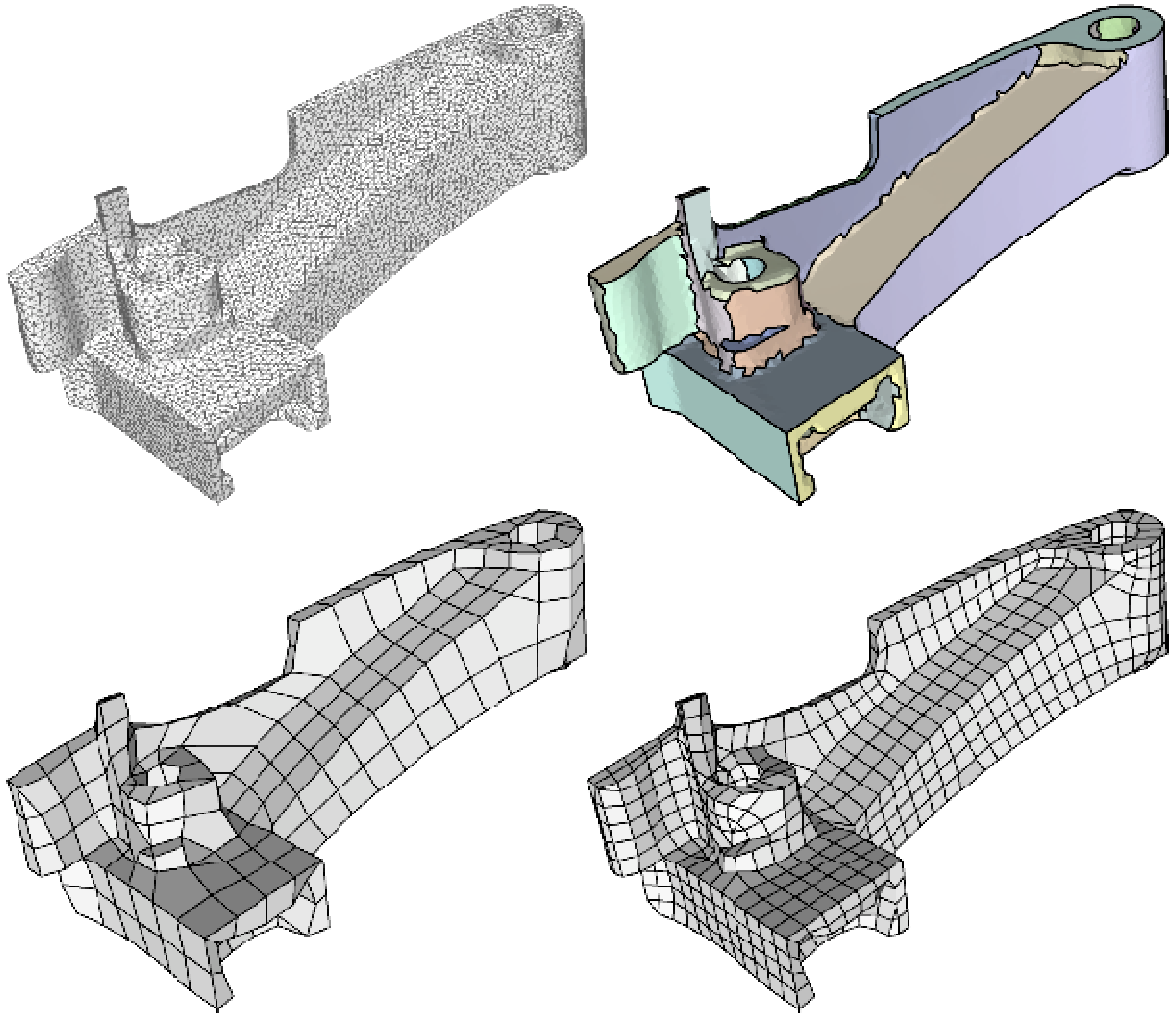


Per-Segment Optimization

- Combinatorial optimization
- Energy functional
  - orthogonality at intersections
  - parallelism within faces

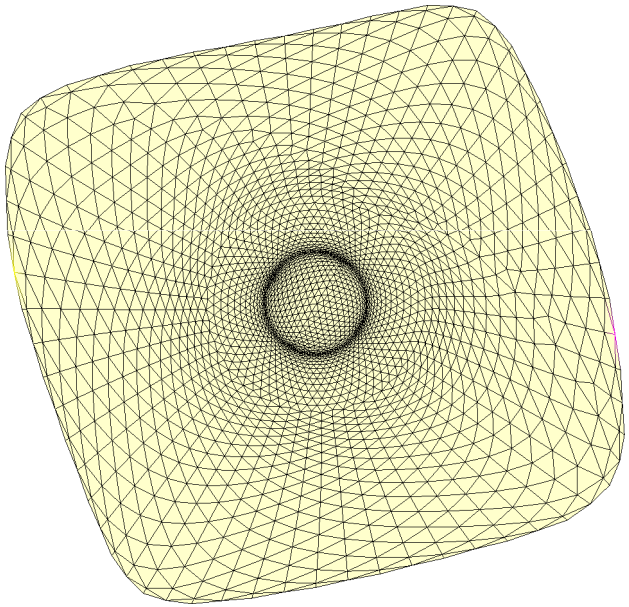


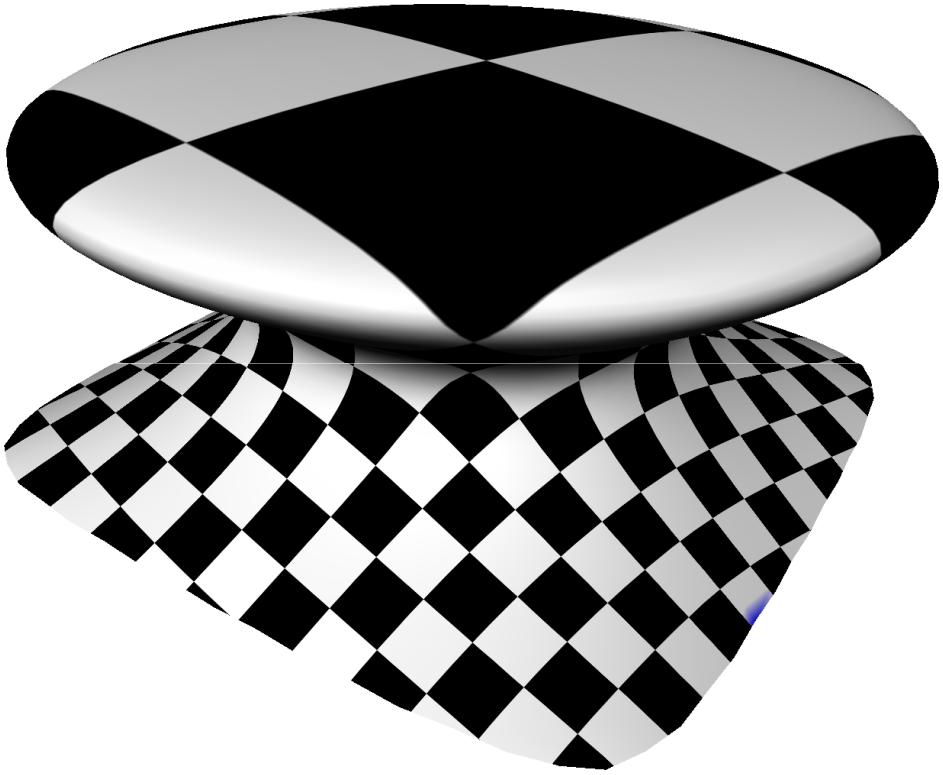
Results



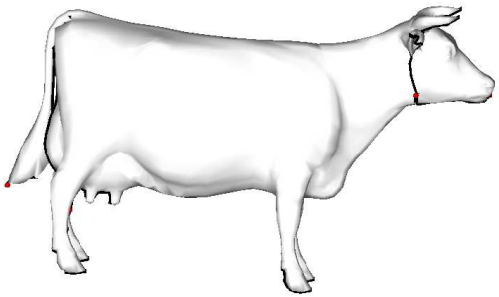
Contouring

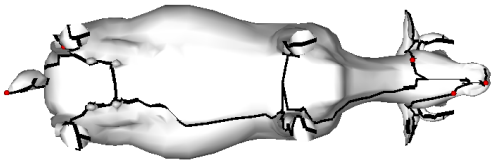
Parameterization (Bruno)



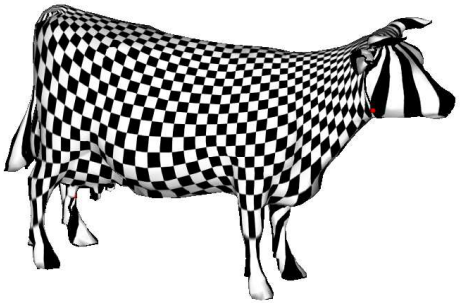


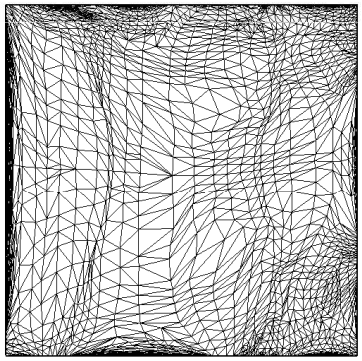
Surface Cutting?



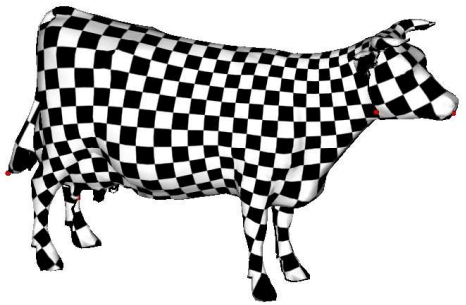


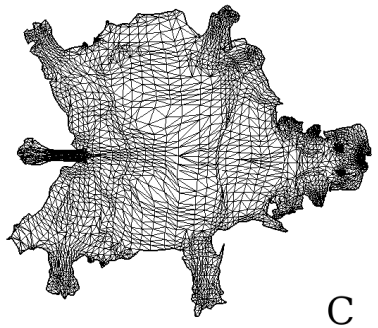
A





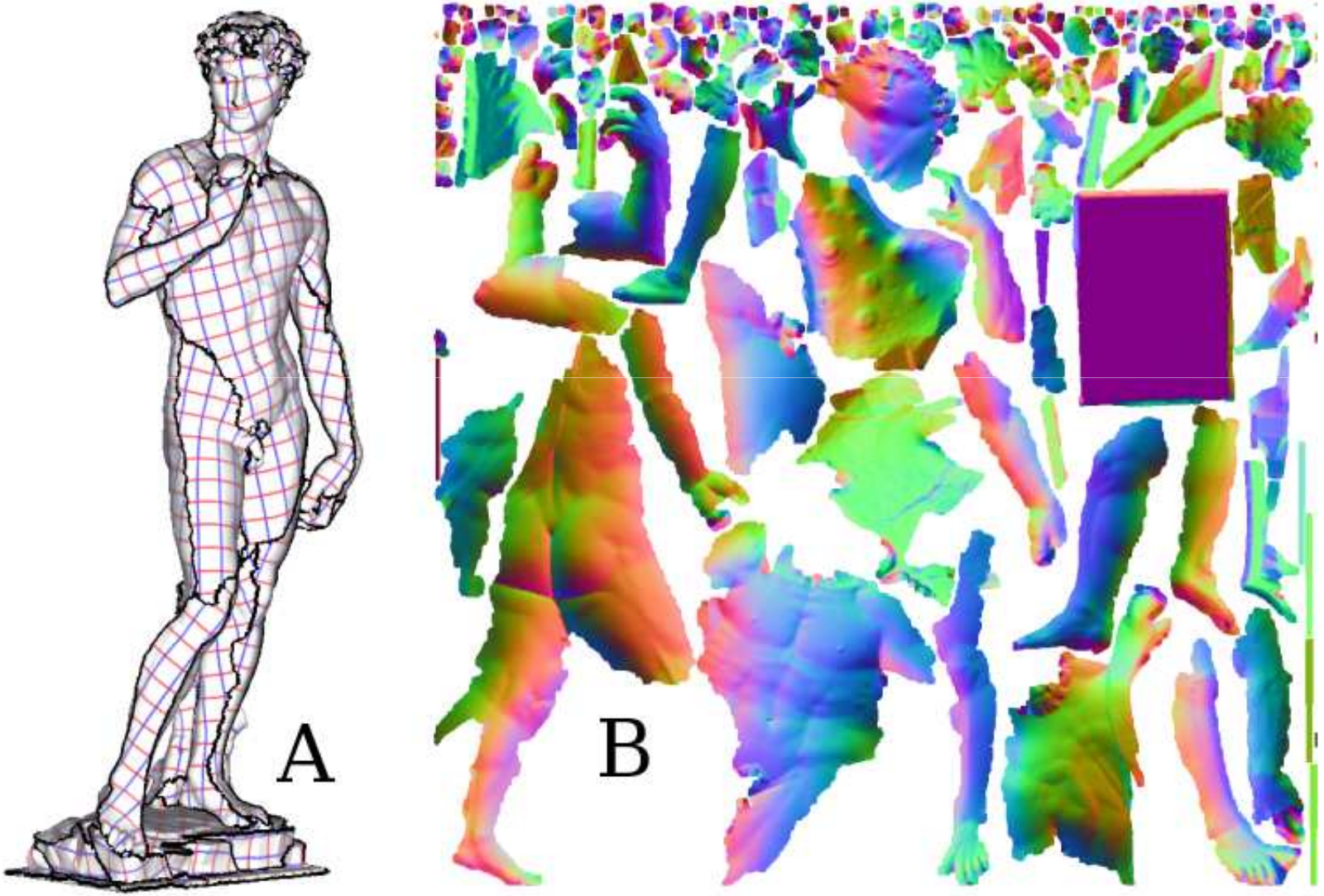
B





C

Area Distortion?



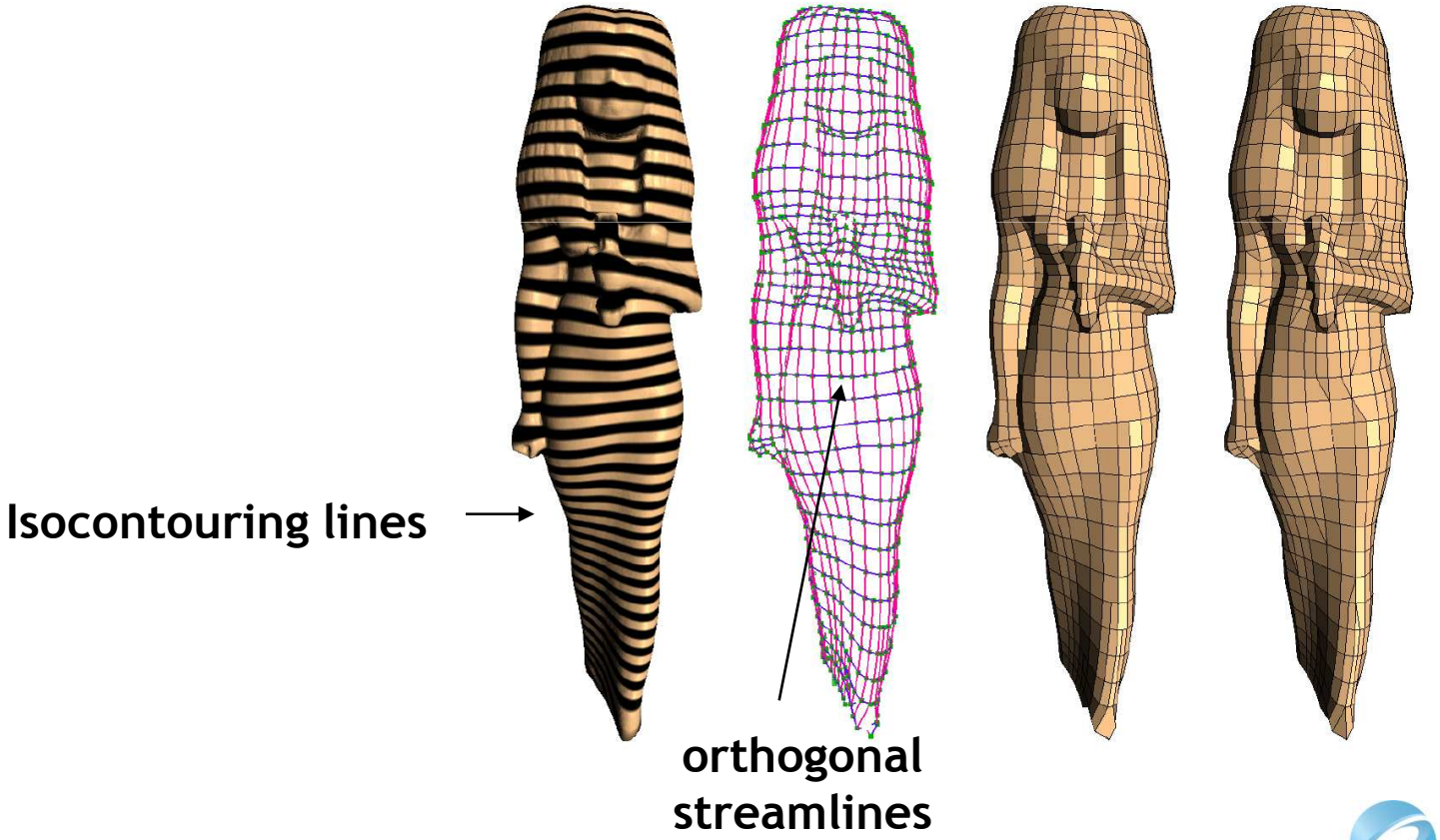
Contouring

Isocontouring lines

Contouring + Streamlines

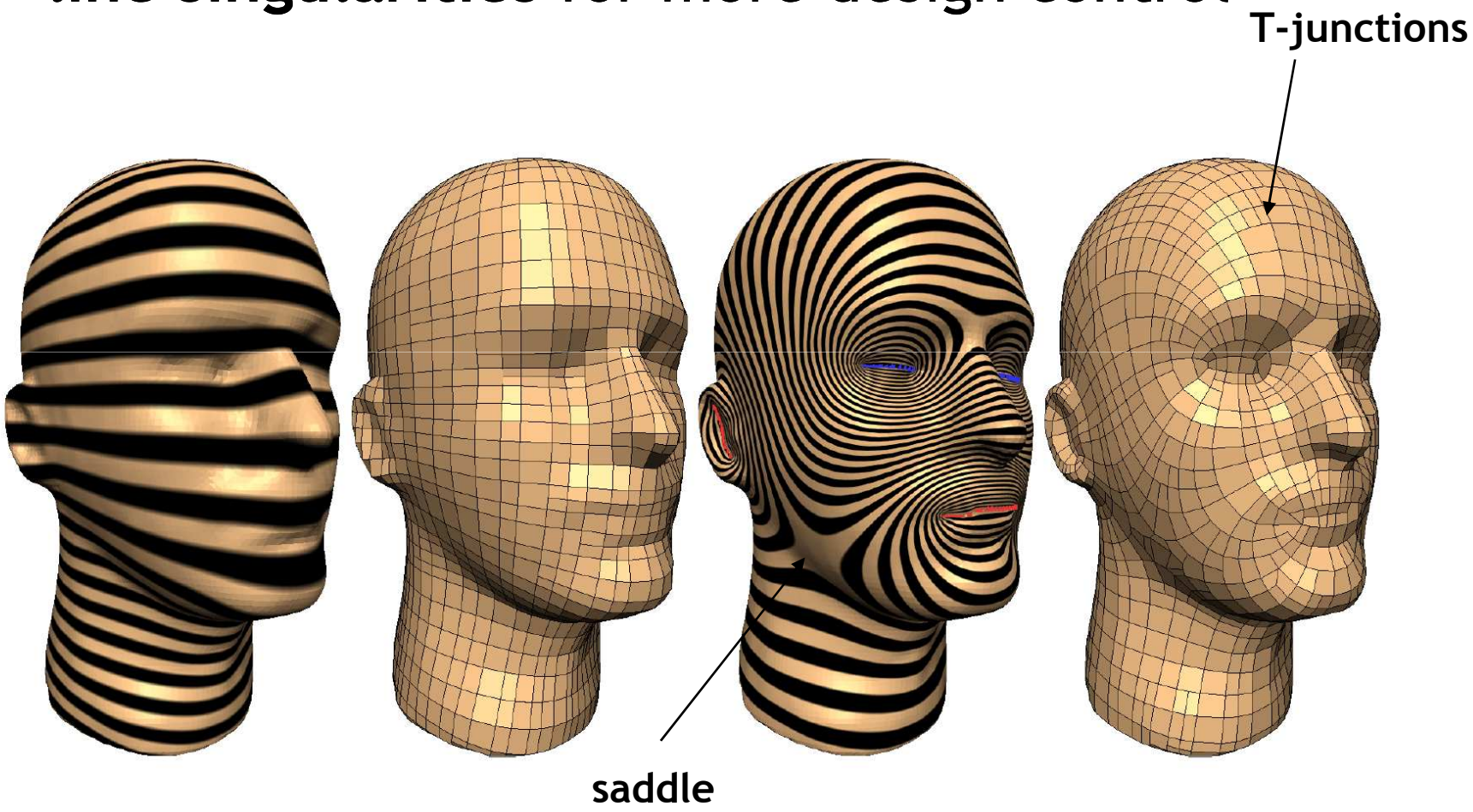
- *Contouring + streamlines*

[Dong, Kircher & Garland 04]



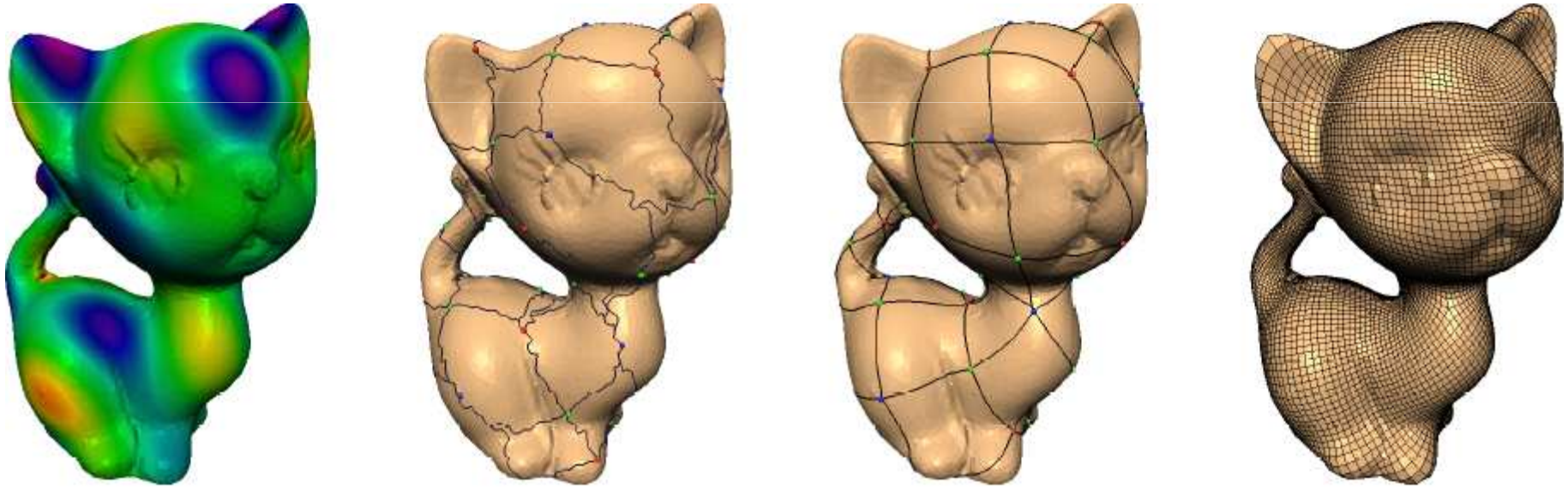
Contouring + Streamlines

- **line singularities** for more design control



Differential Topology

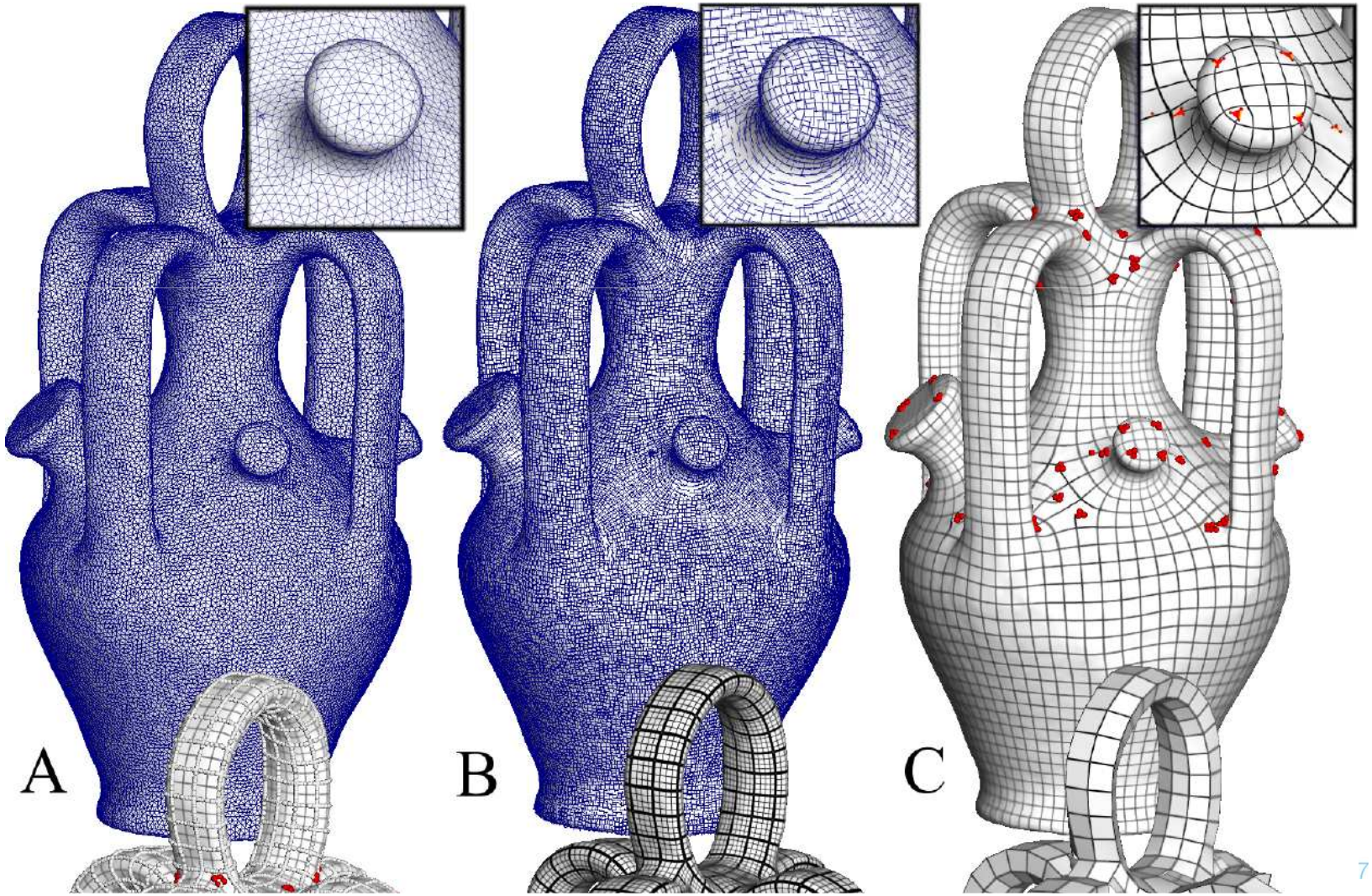
[Dong et al.] SIG 06, *Spectral Surface Quadrangulation*



(a) Laplacian eigenfunction (b) Morse-Smale complex (c) Optimized complex (d) Semi-regular remeshing

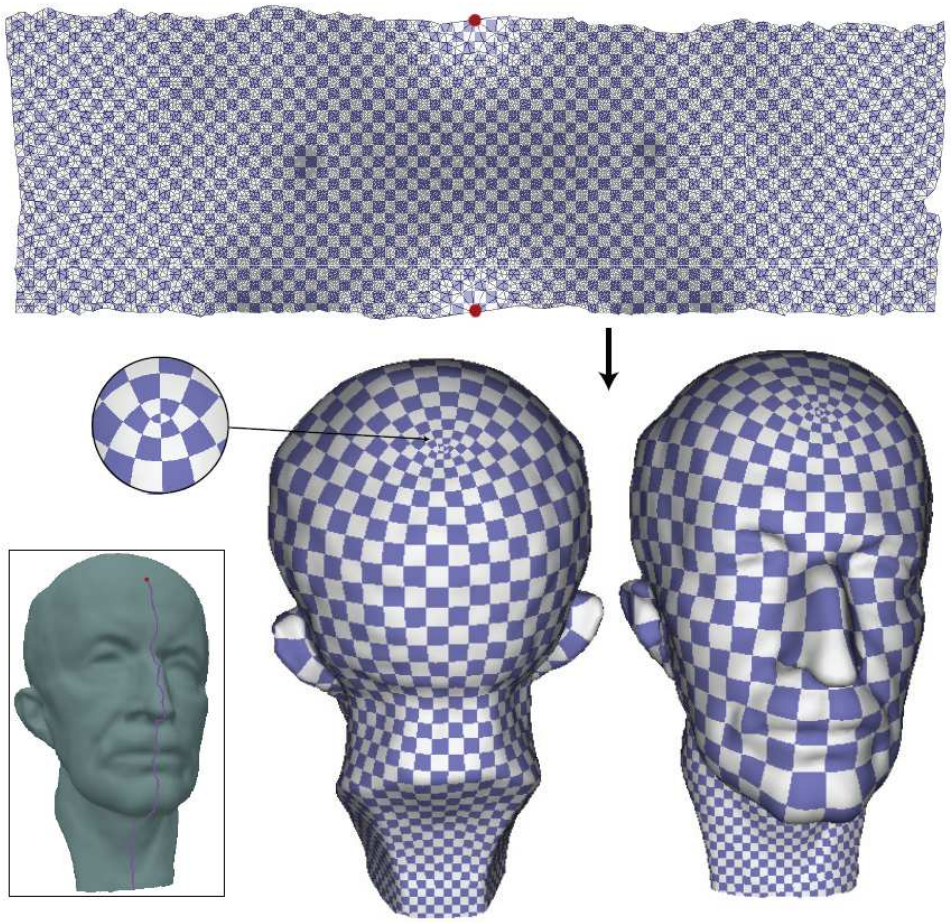
Contouring + Periodic Variables

[Ray et al.] **TOG 06,** *Periodic Global Parameterization*



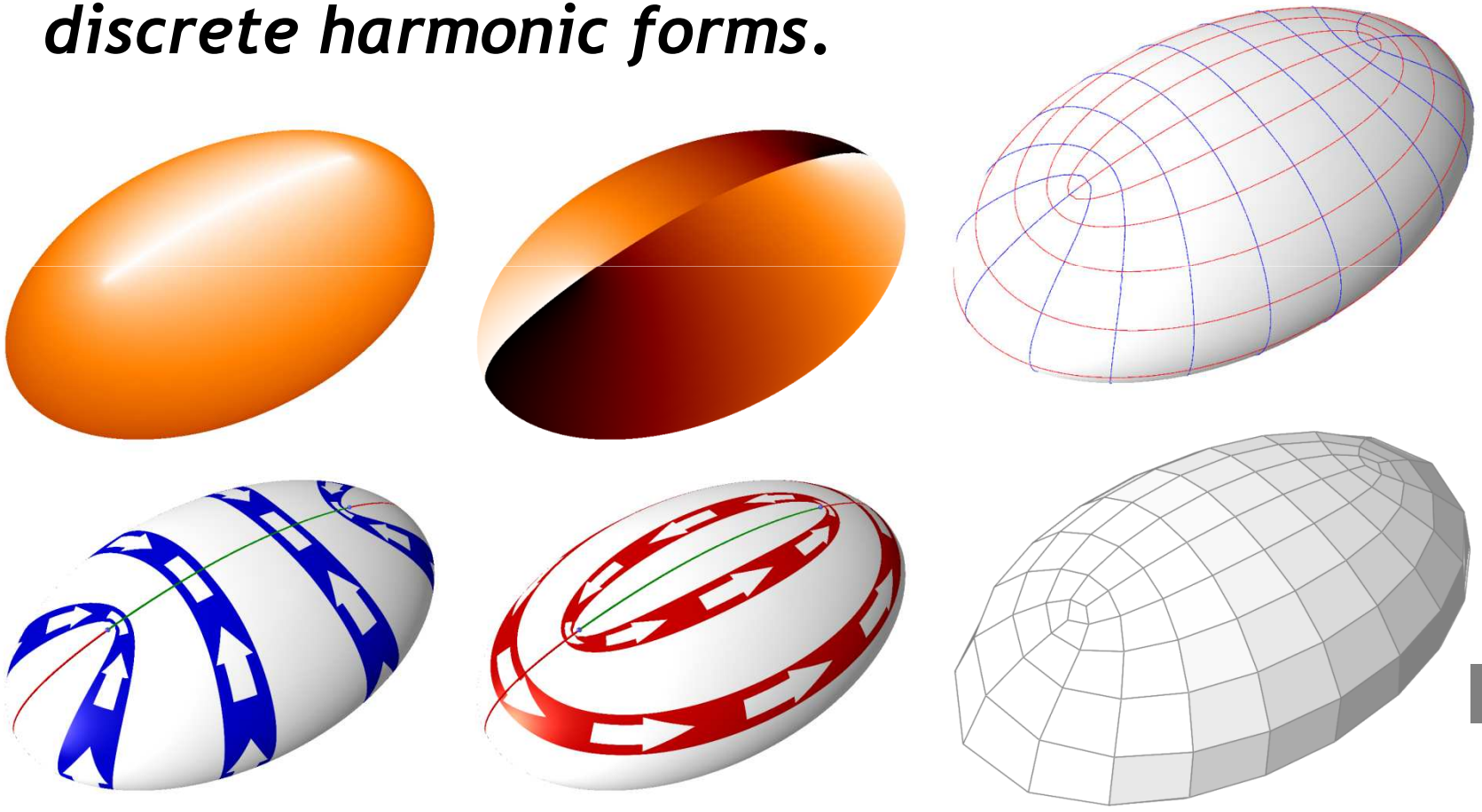
Cone Singularity

[Kharevych, Springborn & Schröder] ACM TOG, *Discrete Conformal Mappings via Circle Patter*



Discrete Harmonic Forms

[Tong, A., Cohen-Steiner and Desbrun], SGP 2006. *Designing quadrangulations with discrete harmonic forms.*

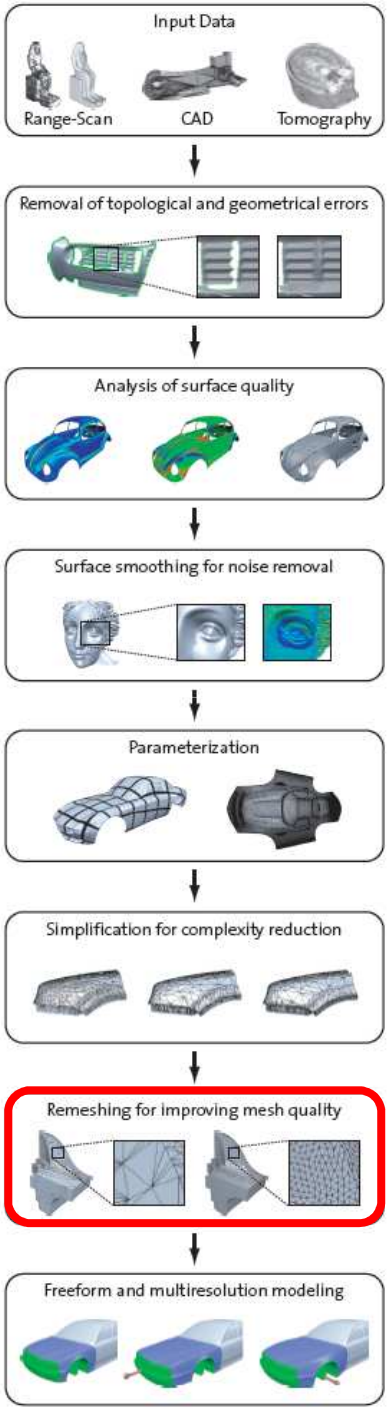


demo

Summary

- **Variety of paradigms**
  - Quadrangle tiling = global problem
- **Contouring**
  - Robust
  - Alignment control through line singularities

- **Future work:**
  - **100% automatic**
  - **Anisotropic**



Error-driven Remeshing

Key Questions

- Error metric?

Key Questions

- Error metric?

- Given #faces
  - what is the best geometric approximant?

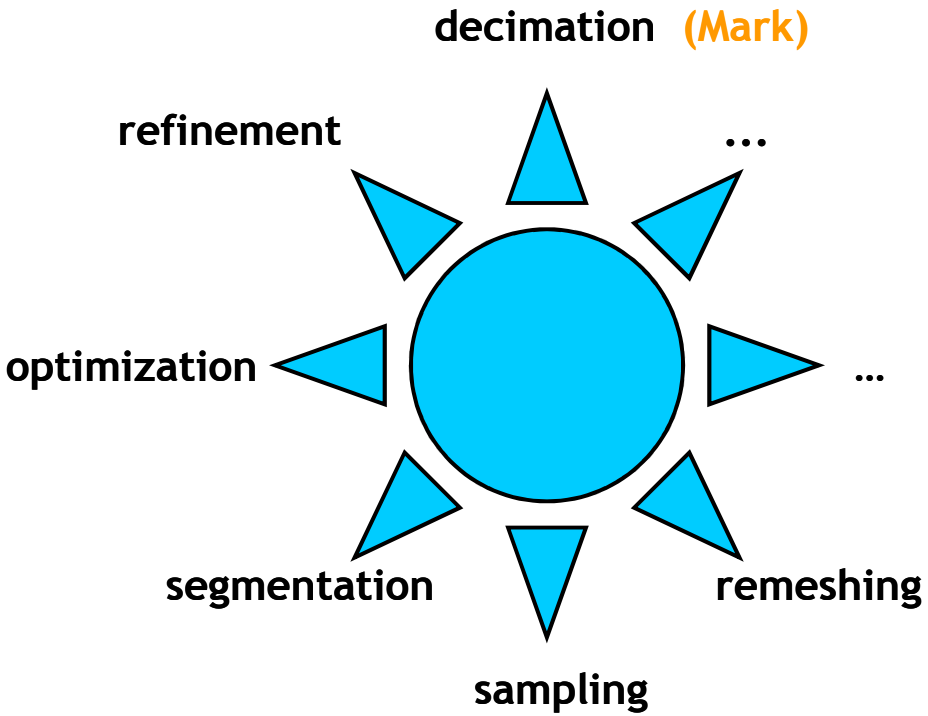
Key Questions

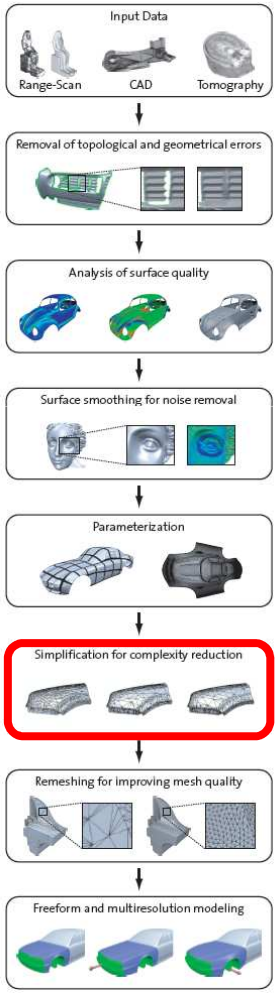
- Error metric?
- Given #faces
  - what is the best geometric approximant?
- Given distortion tolerance
  - smallest mesh approximant below tolerance?

Related Work

Shape approximation:

- central theme in geometry processing

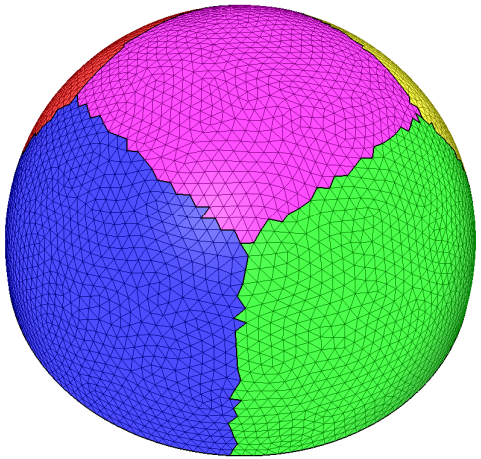


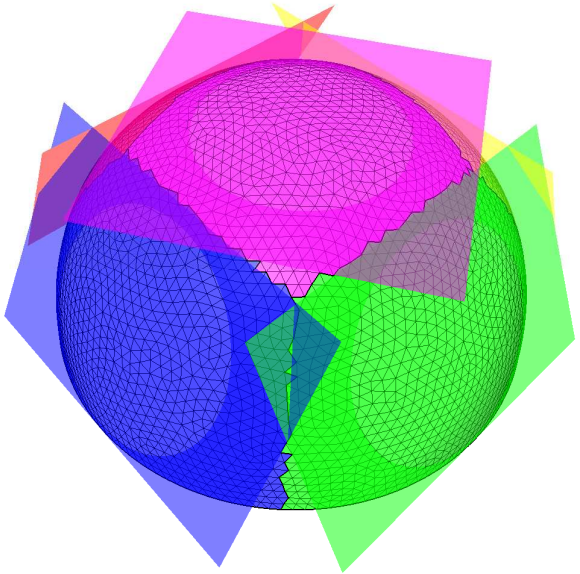


Variational Shape Approximation

[Cohen-Steiner, A. and Desbrun] SIG 2004, ***Variational Shape Approximation***

- We recast surface approximation as a variational k-partitioning problem
  - for each region, find best-fit *linear proxy*
    - "best fit" for a given metric





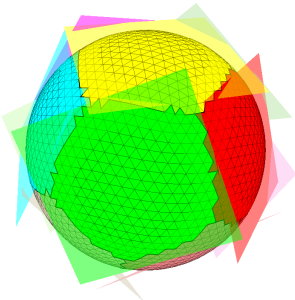
Partition Optimization

Lloyd's clustering as a minimization tool

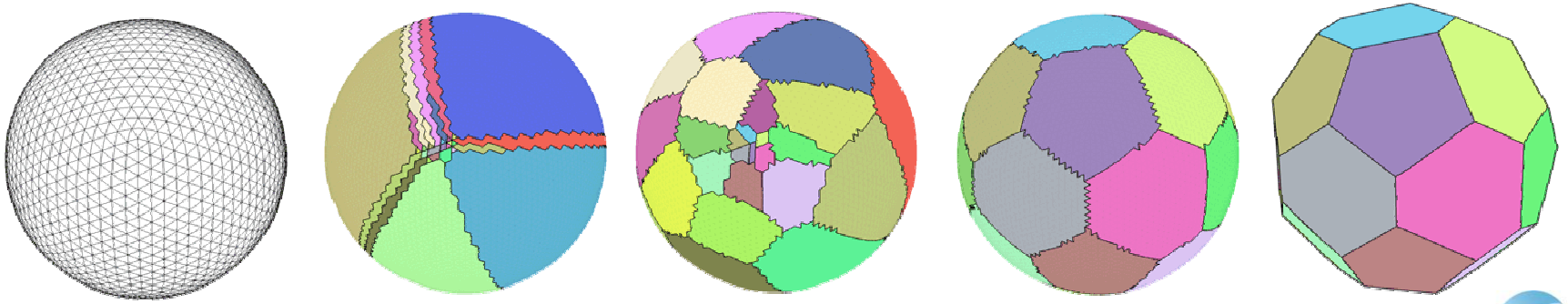
- proxy-based, not point-based
- partition
  - w.r.t. distance to proxies
- find "best fit" proxy
  - equivalent to: move to centroid
- minimizes distortion for a given number of proxies

Algorithm Overview

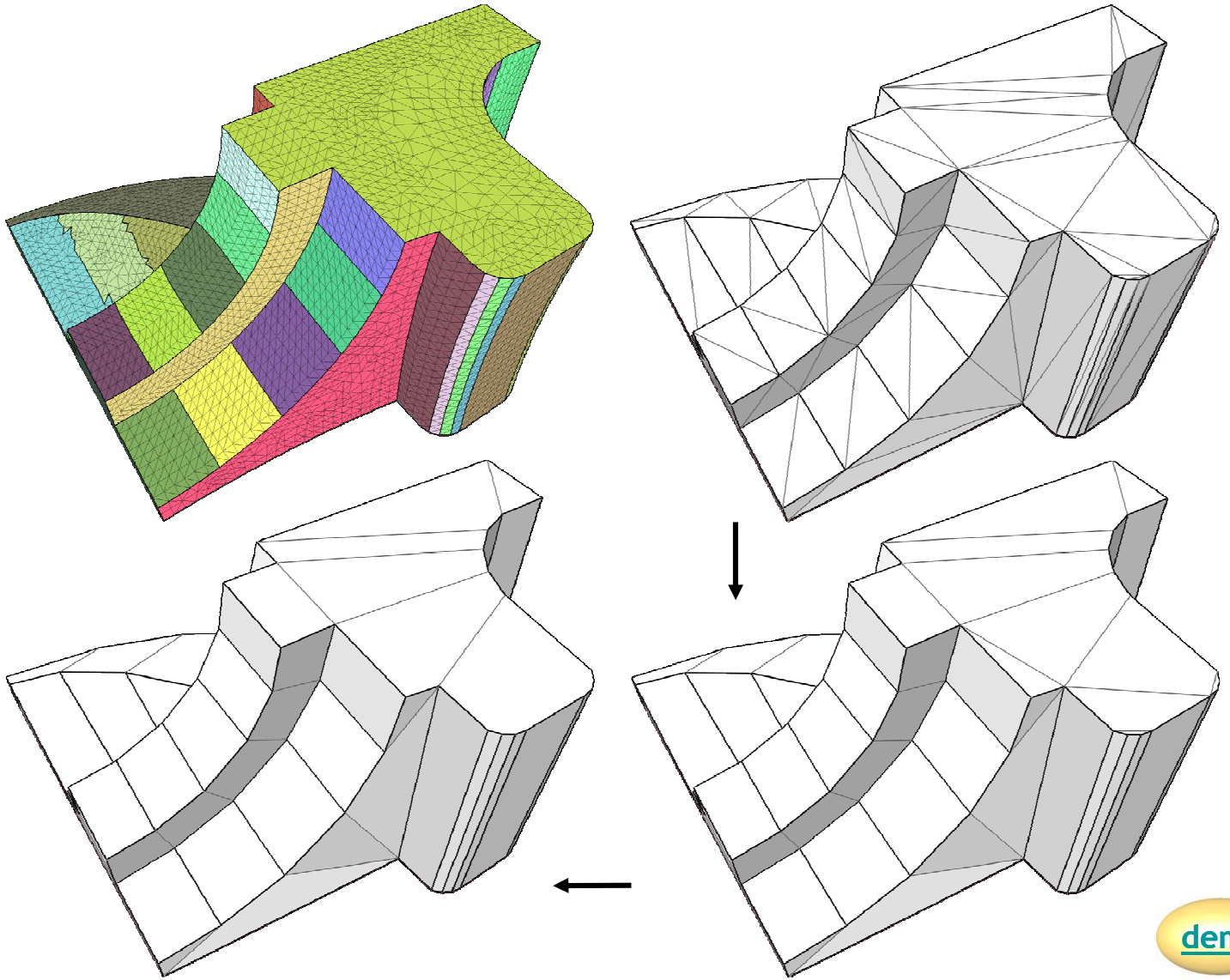
Input: initial mesh, #proxies, metric



1. **Init**
2. **Lloyd-type Iterative Minimization**
   - a: finding new partition
   - b: finding new best fits (proxies) for partition
   - c: possibly, teleport proxies
3. **Meshing**

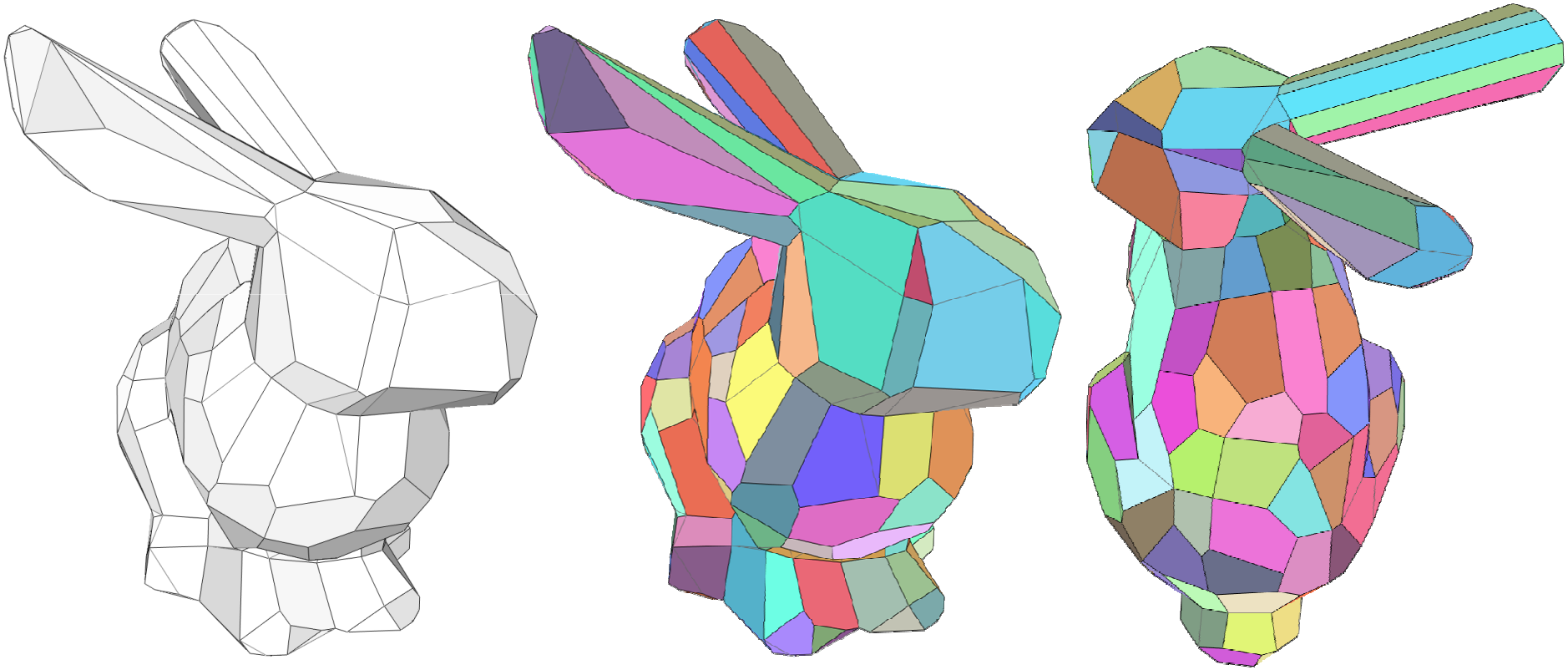


Example: Fandisk



demo

Example: Stanford Bunny

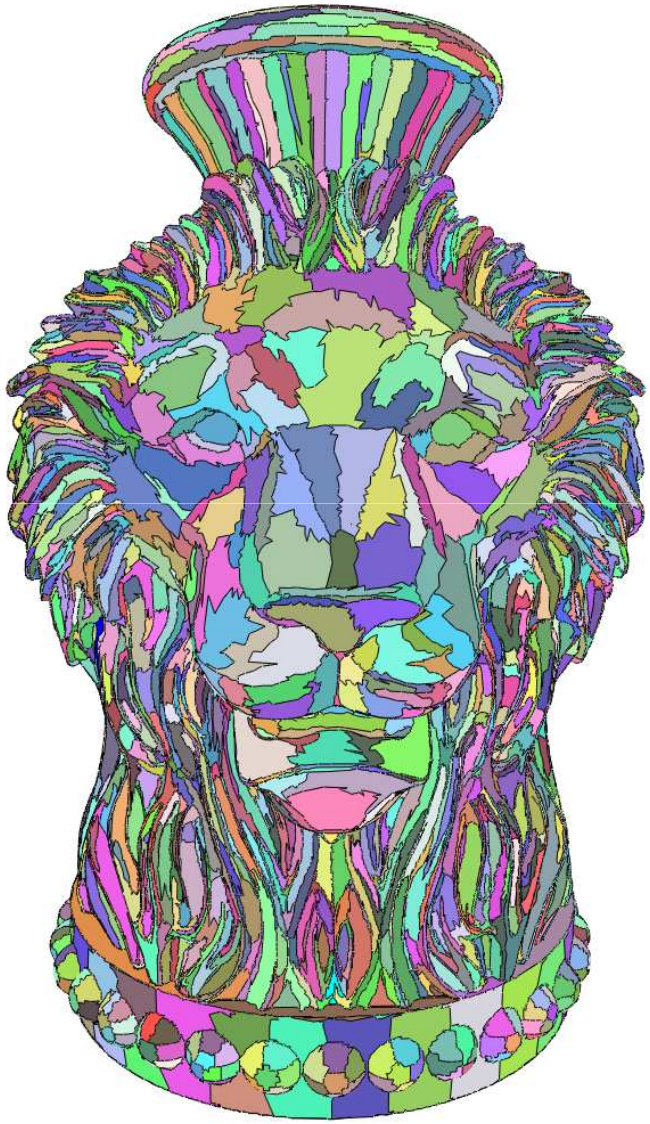


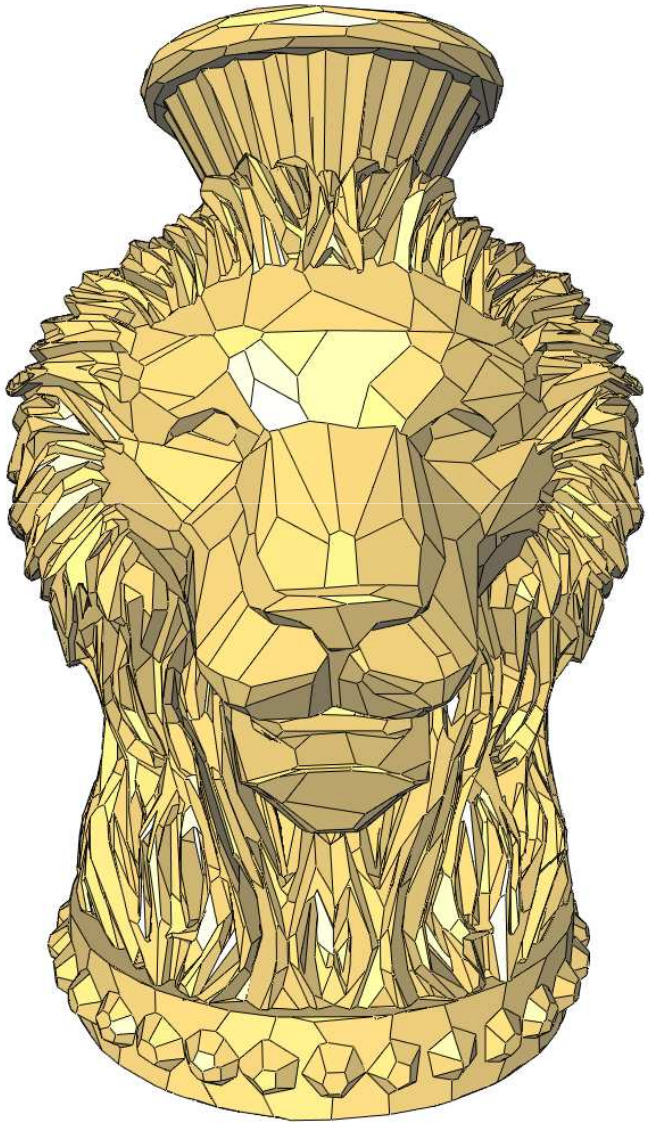
Example: Vase-Lion

400KT

Example: Vase-Lion

Example: Vase-Lion



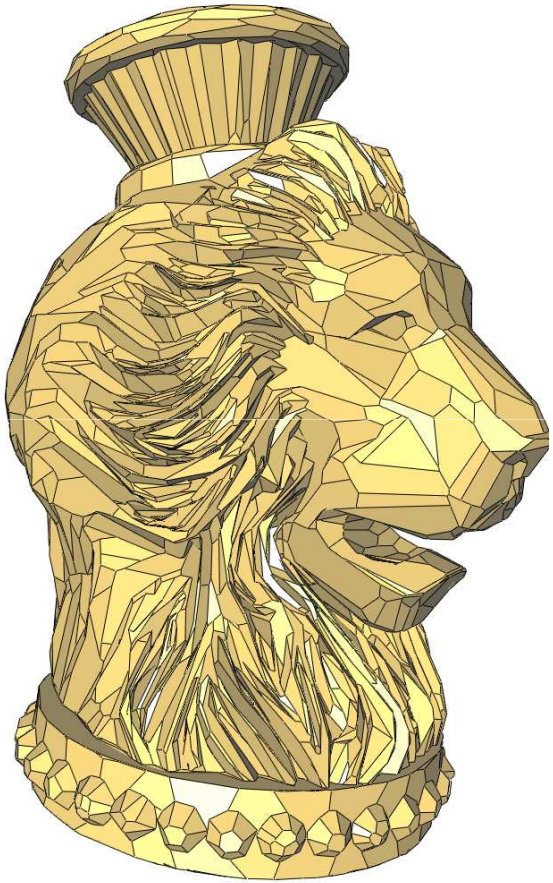


5KP

Example: Vase-Lion

400KT

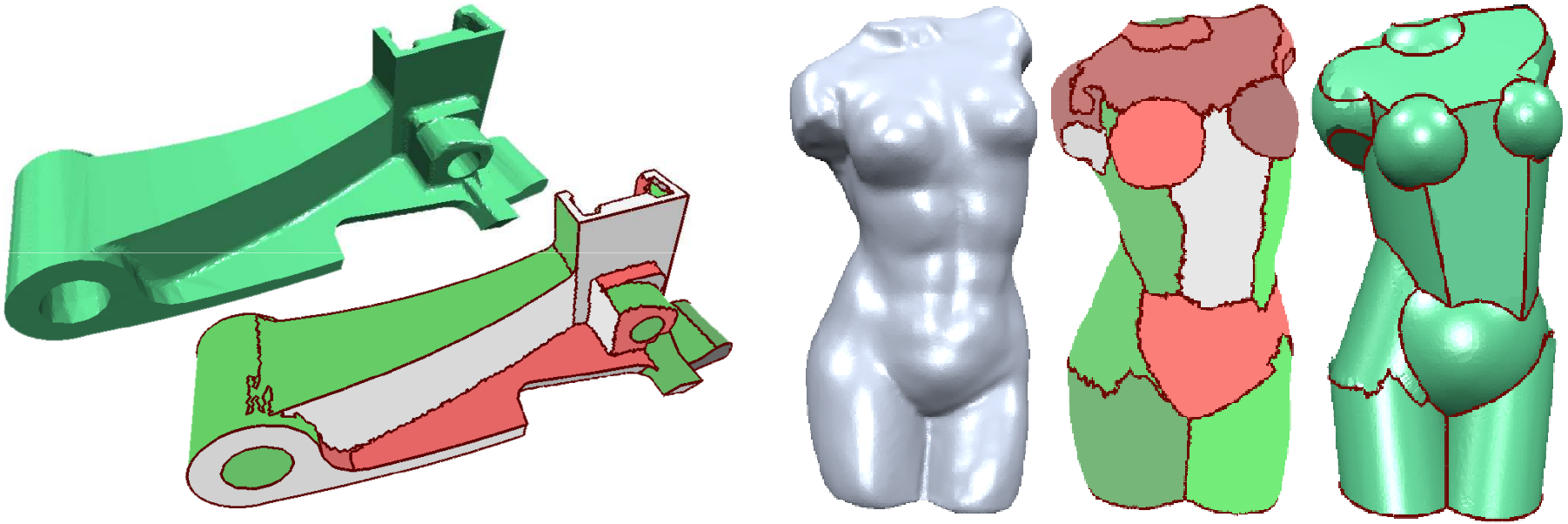
5KP



5KP

Extended to CAD primitives

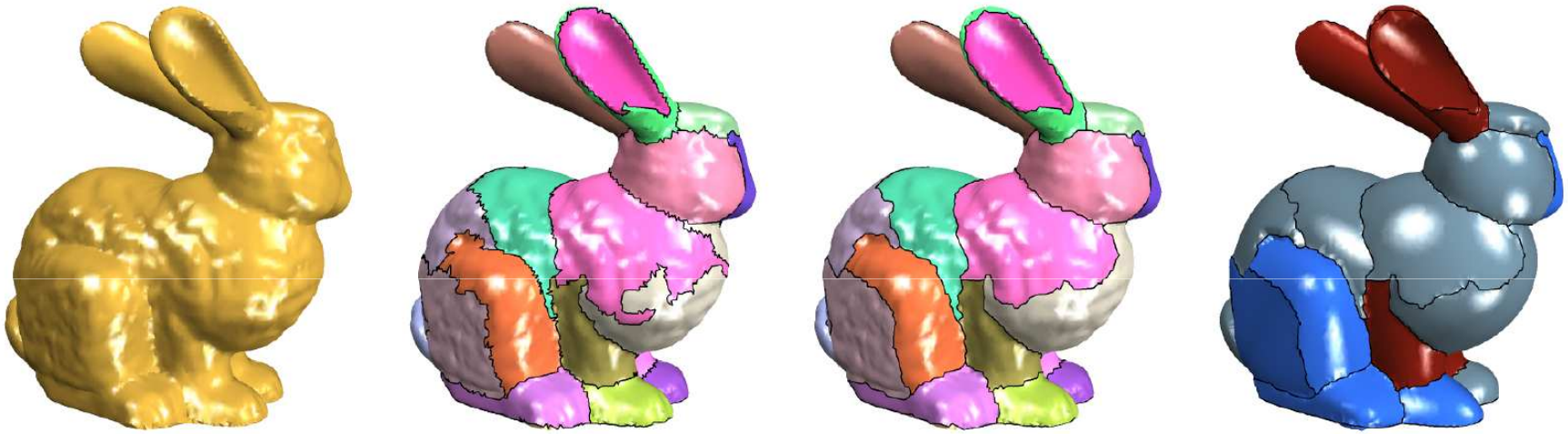
(spheres, cylinders, rolling ball blend patches)



[Wu & Kobbelt] EUROGRAPHICS 05

Structure Recovery via Hybrid Variational Surface Approximation

Extended to Quadrics



[Yan et al. 06] GMP 2006
Quadric Surface Extraction by Variational Shape Approximation

Global Summary

- Remeshing = (re)discretization
- Paradigms: greedy, variational, pliant
- Guarantees of min quality
- …or maximize quality without guarantee
- Control over
  - Shape, orientation, size of elements
- Primitives: points / curves / proxies / volumes?

…or Error driven